Lineshape of the Doppler Broadened 478 keV γ-ray Peak from $^{7*}Li$

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Introduction

Prompt gamma-ray activation analysis (PGA) is one of the most sensitive elemental analysis techniques used in material, life and environmental sciences[1,2]. PGA has widened the applicability of neutron activation analysis. It has come into practical usage, especially for nuclides with large nuclear reaction cross sections. Among the nuclides that benefited from the development of PGA, ^{10}B is one of the highest sensitivities with a 3.837×10^3 barn for its thermal neutron cross section in the $^{10}B(n, \alpha)^{7*}Li$ reaction. A 478 keV γ-ray is emitted on the decay of $^{7*}Li$ to ^{7}Li decays with a halflife of 73 fs (7.3×10^{-11} s)[3]. The halflife of $^{7*}Li$ is so short and comparable to the thermalization time of the $^{7*}Li$ produced by the nuclear reaction that the 478 keV γ-ray experiences a Doppler broadening due to the translational movement of the $^{7*}Li$ nucleus at the time of the emission[4-7]. Previously we discussed the lineshape of the 478 keV γ-ray from $^{7*}Li$ in an analytical form for spectrum fitting in PGA[8]. In this article, we provide a supplementary explanation for the previous publication.

Formulation

The $^{7*}Li$ particles produced via the $^{10}B(n, \alpha)$ reaction travel with an initial

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velocity (v_0) of 4.8 x10^6 ms^{-1} without any preferential direction. We place a detector on the z-axis. The velocity along the line-of-sight (v_z) of the ^{7*}Li immediately after production is $v_0 \cos\theta$ when we take the z-axis as the line-of-sight (Fig. 1). ^{7*}Li particles moving in the directions with the same θ have the same v_z; the fraction of the ^{7*}Li having the same Doppler shift is

$$2\pi v_0 \sin\theta(-d\theta)/4\pi v_0^2 = \sin\theta(-d\theta)/2v_0 \qquad (1)$$

considering the solid angle. Since $v_0\sin\theta(-d\theta)=d(v_0\cos\theta)=dv_z$, the fraction is constant for all line-of-sight velocity values independent of v_z for $|v_z|<v_0$. This constancy corresponds to the fact that the surface area of a sphere cut by two parallel planes is proportional to the distance between the planes independent of the cutting site.

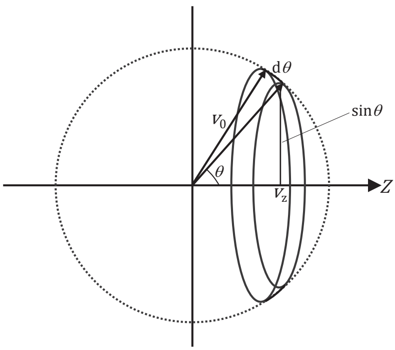

Figure 1. Projected velocity distribution of isotropically emitted ^{7*}Li.

Let us consider the case where the slowing down of ^{7*}Li is expressed by the exponential time dependence with a degradation constant D,

$$v=v_0 e^{-Dt} \qquad (2)$$

At time t, ^{7*}Li gives off γ-rays at a rate proportional to $\lambda n_0 e^{-\lambda t} dt$, where t is the elapsed time from the production of ^{7*}Li, λ is the decay constant of ^{7*}Li (ln2 / 73 fs), n_0 is the number of ^{7*}Li at t=0. The number of γ-ray photons $H(E)$ at the

Doppler shift E, which corresponds to the number of the photons $f(v_z)$ at the velocity of the line-of-sight v_z, is written as

$$H(E)\mathrm{d}E = \varepsilon \int_{t=0}^{t=t_E} \frac{c}{2E_0 v_0} \lambda n_0 e^{-\lambda t} \mathrm{d}t\mathrm{d}E \qquad (3)$$

where ε is a normalization factor including detection efficiency, E_0 is the unshifted γ-ray energy of 478 keV, c is the velocity of light. The upper bound of the integral t_E is determined by the maximum Doppler shift E_m for v is $E_0 v/c$. The velocity changes as $v=v_0 e^{-Dt}$ (eq. 2). The maximum E_m changes with time as

$$E_M = E_0 \frac{v_0 \mathrm{e}^{-Dt}}{c} \qquad (4).$$

The upper bound of the integral (3) should be

$$\frac{1}{D} \ln \frac{E_0 v_0}{c E_M} = t_E \qquad (5).$$

Integrate by t we obtain

$$H(E) = \frac{\varepsilon c n_0}{2E_0 v_0} \frac{\lambda}{\lambda - D} \left[1 - \left(\frac{cE}{E_0 v_0} \right)^{\frac{\lambda - D}{D}} \right] \qquad (6).$$

This equation is identical to the equation (12) of reference x.

Conclusion

We have derived the analytical expression of the Doppler broadened 478 keV γ-ray lineshape from ^{7*}Li produced via the ^{10}B(n, α)^{7*}Li reaction. Enhancement of the computation power of personal computers in the past 20 years has made it possible to fit PGA data to the equation (6) in less than a second. As the next step, we plan to incorporate the Doppler broadened 478 keV γ-ray fitting procedure in a routine γ-ray spectrum analysis software.

References

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