A contract curve model of working hours and geometric mean regression analysis

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Abstract

The framework of supply curve of working hours is a main tool to explain how working hours are determined or how working hours respond to tax rate changes. However, the estimation task of the supply curve is not complete yet.

We constructed a new model of working hours incorporating both supply and demand curves of working hours. The constructed model demonstrates that (1) working hours (h) and wage earnings (E) are determined jointly at an equilibrium point on a contract curve where demand and supply of workers are equal, and (2) the contract curve passes through the intersection of the supply and demand curves of working hours.

These results imply that (1) it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours, and (2) a contract curve of working hours should be estimated to measure the wage elasticity of working hours.

For the estimation of contract curves, geometric mean regression (GMR) is selected for three reasons. First, the constructed model is a type of measurement error model. That is, both variables (h and E) in the contract curve equations are measured with errors. Second, the GMR estimator is the maximum likelihood estimator. Third, the coefficient of determination (R^2) of both variables is equal in the GMR.

The estimated wage elasticity of working hours for four industry groups is between −0.13 and −0.23. The transportation group has a slightly higher elasticity than the others, and high school graduates have a slightly higher elasticity than college graduates.

In GMR, there is a simple relation between the coefficient of determination (R^2) and correlation coefficient (r_{xy}) as follows: $R^2 = (1+|r_{xy}|)/2$.

Keywords : Contract curve, Working hours, Wage elasticity, Geometric mean regression, Deming regression, Measurement error model

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1. Introduction

The framework of supply curve of working hours is a main tool to explain how working hours are determined or how working hours respond to tax rate changes. Thus, many empirical articles have been written on this topic. However, the estimation task is not yet complete.

Keane (2011) and Bargain and Peichl (2013) argued that there is no clear consensus on the magnitude of wage elasticity. In addition, Pencavel (2016) noted that most studies have neglected the identification problem or the employer's role.[1]

As suggested by Pencavel (1986, 2016) , this study constructs a new model that incorporates both workers and firms. The constructed model demonstrates that working hours (h) and wage earnings (E) are determined jointly at an equilibrium point on a contract curve where the demand and

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supply of workers are equal. Consequently, we argue that a contract curve should be estimated to measure the wage elasticity of working hours.

In the estimation, we apply geometric mean regression (GMR), which is a special case of errors-in-variables regression or Deming regression. We choose this method because both variables (h and E) are measured with errors and the constructed model is a type of measurement error model.

The remainder of this paper is organized as follows: Section 2 presents a basic model of working hours. In Section 3, the GMR is explained. Then, by applying GMR, we estimate the contract curve of working hours and wage elasticity. Finally, Section 4 presents our concluding remarks.

2. Basic model

The basic model incorporates both the supply and demand functions of working hours.

It demonstrates that working hours (h) and wage earnings (E) are determined jointly at an equilibrium point on a contract curve.[2]

2.1. Model assumptions

The assumptions of the model are ordinary and are the following:

(i) Workers' utility function is U(E, h), where E and h denote wage earnings and working hours, respectively. U(E, h) is quasi-concave, and $U_E > 0$ and $U_h < 0$.

(ii) Firms have the production function AF(L, h), where A is total factor productivity and L is the number of employees. The capital stock is constant.

(iii) In the labor market, the equilibrium wage rate is determined to equate the demand and supply of workers.

2.2. Supply and demand functions of working hours

To clarify the model, we assume an example of indifference and isoprofit curves. This assumption does not alter the essence of the model.

2.2.1. Supply function of working hours

A quasi-concave utility function of a worker is assumed as shown in Equation (1):

$$U(E, h)=E-\alpha(h+\beta)^2 , \quad (1)$$

where α and β are positive parameters ($\alpha>0$, $\beta>0$). On an indifference curve, $dE/dh>0$ and $d^2E/dh^2>0$. This indicates that the marginal rate of substitution is positive and increasing. The utility-maximizing behavior of a worker is formulated as follows:

$$\text{Max } U(E, h)=E-\alpha(h+\beta)^2 \quad \text{st. } E=wh,$$

where w is the wage rate. The first-order condition is $dU/dh=w-2\alpha(h+\beta)=0$, from which we have the following supply function of working hours: $w=2\alpha(h+\beta)$. Multiplying both sides of the equation by h, we obtain the supply function of working hours in the h-E plane as follows:

$$E=2\alpha(h^2+\beta h) . \quad (2)$$

The supply curve passes through the origin (Fig. 1).

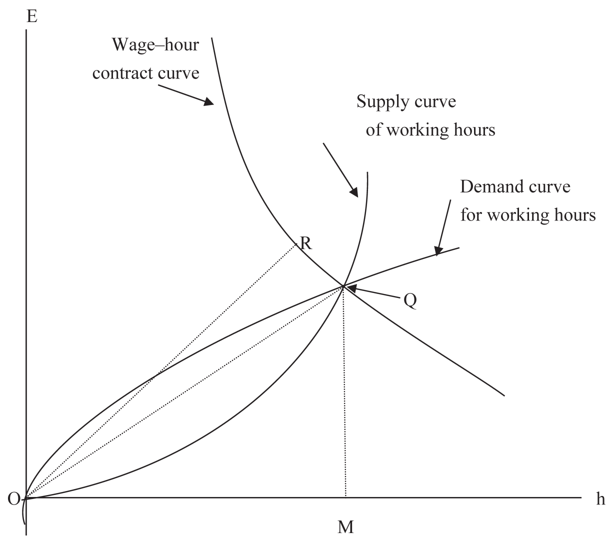

Fig. 1 Equilibrium of a firm and its workers

2.2.2. Demand function for working hours

We assume that the isoprofit curve in Equation (3) is derived from the production function AF(L, h):[3]

$$k=h-\gamma(E+\delta)^2, \quad (3)$$

where k indicates the profit level and γ and δ are positive parameters ($\gamma>0$, $\delta>0$). On an isoprofit curve, $dE/dh>0$ and $d^2E/dh^2<0$. This indicates that the marginal productivity of working hours is positive and diminishing. The demand function for working hours is derived from a firm's profit-maximizing behavior, which is formulated as follows:

$$\text{Max } k=h-\gamma(E+\delta)^2 \quad \text{st. } E=wh.$$

The first-order condition is $dk/dh=1-2\gamma w(wh+\delta)=0$, from which we obtain the following demand function for working hours: $1=2\gamma w(wh+\delta)$. Multiplying both sides of the equation by h, we obtain the demand function for working hours in the h-E plane as follows:

$$h=2\gamma(E^2+\delta E). \quad (4)$$

The demand curve passes through the origin (Fig. 1).

2.3. Equilibrium of a firm and its workers

Fig. 1 illustrates the demand and supply curves for working hours in a firm and its workers. These two curves are parabolas intersecting at Q, where the wage rate is ∠QOM, and the desired working hours of a firm and its workers coincide.

Is equilibrium realized at Q? The condition for equilibrium is that the demand and supply of workers should be equal at the wage rate (Rosen, 1969, p.261). This is possible only when ∠QOM is equal to the market wage rate by chance. However, there is no assurance that this is true.

If the market wage rate is ∠ROM (>∠QOM), there will be an excess demand for workers at Q. Then, the equilibrium point moves up along the contract curve, which is the locus of the tangency points of the indifference and isoprofit curves. (The contract curve passes through point Q.) Thus, R becomes the equilibrium point for the firm and its workers.

In summary, the equilibrium point lies somewhere on the contract curve where the demand and supply of workers are equal. We name the contract curve the wage–hour (WH) contract curve. The result implies that it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours. In the example in the previous section, the WH contract curve is shown in Equation (5).[4]

$$4\alpha\gamma(h+\beta)(E+\delta)=1. \quad (5)$$

2.4. Market equilibrium of an industry

We consider an industry where all firms have the same production function (or isoprofit curves) and all workers have the same utility function (or indifference curves). The industry's market equilibrium is described by three endogenous variables (E, h, and L) and the following three equations:

$$AF_L(L, h)=E+C \quad (6\text{-}1)$$

$$L=L^s(E/h) \quad (6\text{-}2)$$

$$U_t(E, h)/U_E(E, h)=Y_t(E, h)/Y_E(E, h). \quad (6\text{-}3)$$

Equation (6-1) is the demand function for workers in implicit form. Equation (6-2) is the supply function of workers, which is an increasing function of the wage rate (E/h). Equation (6-3) shows the WH contract curve in its general form. Here, $Y(E, h)=k_1$ and $U(E, h)=k_2$ are the representative firm's isoprofit curves and the representative worker's indifference curves, respectively (k_1 and k_2 are parameters).

By inserting Equation (6-2) into (6-1), we find that $AF_L(Ls(E/h), h)=E+C$, which is the equation of the workers' market equilibrium (WM equilibrium curve). As shown in Fig. 2, this curve has a positive slope. Thus, the market equilibrium is at point R, which is the intersection of the WM equilibrium curve and the WH contract curve.[5]

The WM equilibrium curve shifts downward if the supply of workers to this industry increases. Then, the equilibrium point moves from R to R' along the WH contract curve, and the wage rate decreases.

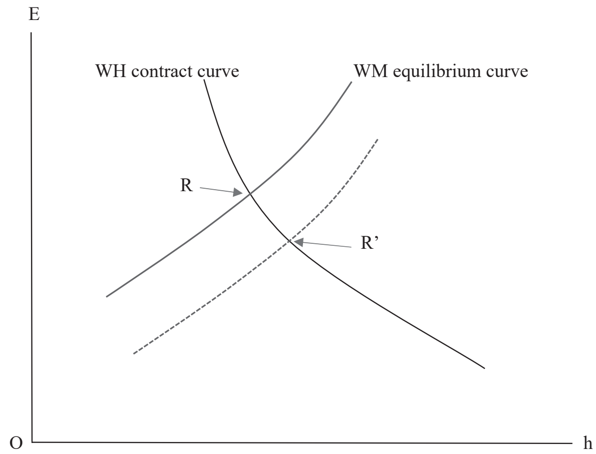

Fig. 2 Market equilibrium of an industry

2.5. Tax effects

The effect of wage income tax on working hours is either positive or negative. If the supply curve of working hours is positively sloped, taxation decreases equilibrium working hours. Conversely, if the supply curve is negatively sloped (backward-bending supply curve), taxation increases equilibrium working hours.

First, we explain the case of a positively sloped supply curve of working hours. When the tax rate is $\tau(1>\tau>0)$, the net wage rate decreases to $(1-\tau)w$. Therefore, for a given wage rate, the desired supply of working hours decreases. Hence, the supply curve shifts to the left (Fig. 3). Accordingly, the intersection of the supply and demand curves moves leftward from Q to Q'. As the WH contract curve passes over Q', the WH contract curve also shifts leftward from RQ to R' Q'. Thus, the market equilibrium moves leftward from R to R' along the WM equilibrium curve. In summary, if the supply curve of working hours is positively sloped, wage income tax decreases the equilibrium working hours.

Second, in the case of a backward-bending supply curve, the tax effects work in opposite directions. Income tax shifts the supply curve rightward, and the intersection of the supply and demand curves moves rightward. Accordingly, the WH contract curve shifts rightward, and the market equilibrium moves rightward along the WM equilibrium curve. In summary, in the case of a backward-bending supply curve, wage income tax increases the equilibrium working hours.

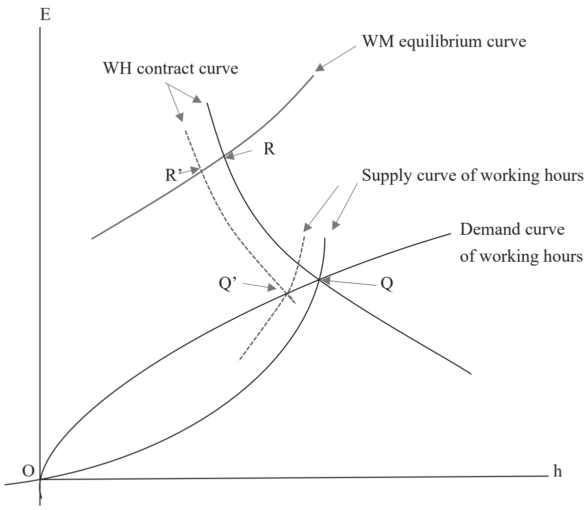

Fig. 3 Tax effects

3. Estimation of contract curves of working hours

3.1. Strategy

We consider a group of manufacturing industries. Assume that all industries have the same production function as AF(L, h) with various levels of A. Additionally, we assume that all workers have the same preference (indifference curves). We then show that the equilibrium point of each industry lies on a common WH contract curve.

Note (3) shows that the isoprofit curve of a firm whose production function is AF(L, h) is the solution E(h) of the differential equation. A (total factor productivity) did not appear in the differential equation. Therefore, industries (firms) having the same production function with various levels of A have the same isoprofit and demand curves for working hours. Thus, if their workers share the same preference, they have a common WH contract curve. The equilibrium point of an industry with a larger A is located at a higher point on the common WH contract curve.

To summarize, we first take a group of industries that are supposed to have similar production functions. Second, we restrict the workers to a group that is supposed to have the same preference. Finally, using their data on wage earnings (E) and working hours (h), we can estimate the WH contract curve.

3.2. Data

We employed the Basic Survey on Wage Structure (BSWS), which is conducted every June by the Japanese Ministry of Health, Labour and Welfare (Ministry of Health, Labour and Welfare of Japan). The BSWS is a type of industry survey. As employers responded to the questionnaire, we consider data on working hours and wage earnings to indicate equilibrium points on their WH contract curves.

The survey classified 90 industries and differentiated firms by their number of employees: (1) 1,000 or more employees, (2) 100–999 employees, and (3) 10–99 employees. Additionally, the survey indicated workers' educational background (i.e., college or high school graduates), gender (i.e., male or female) and age.

The scheduled working hours (SWH) and overtime working hours (OTH) were selected from the BSWS. The former are standard working hours, as stipulated by office regulations. Therefore, h=SWH+OTH is total working hours. For wage earnings (E), contractual cash earnings (CCE) and annual special cash earnings (ASE) are selected. The CCE includes payments for both SWH and OTH in June of the survey year. ASE is the previous year's annual bonus payments. Thus, we calculate the monthly wage earnings (E) as E= CCE+ASE/12.

Fig. 4 presents plots of 14 manufacturing industries with male workers, college graduates, and those aged 50–54. We assume that the industries had

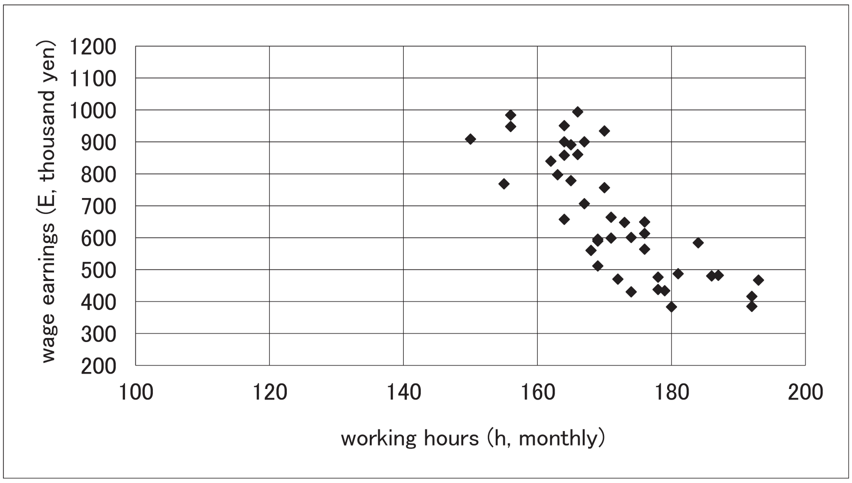

Fig. 4 Working hours (h) and wage earnings (E) in fourteen manufacturing industries (male, college graduate, age 50-54, 2015)

similar production functions and that their workers had similar preferences. The vertical axis denotes wage earnings (1,000 yen), and the horizontal axis denotes monthly working hours. We estimate the WH contract curve using a regression analysis of the plots.[6]

3.3. Geometric mean regression

Geometric mean regression (GMR) is used, which is a special case of errors-in-variables regression or Deming regression. We cannot use ordinary least squares (OLS) because both working hours (h) and wage earnings (E) are measured with errors.[7]

Errors-in-variables regression is explained as follows: In the contract curve model, $E=\alpha+\beta h$ is assumed to be a linear relationship between the two variables. The two observed variables (x_i, y_i), i=1…n, have errors that may be written as follows:

$x_i=h_i+e_{xi}$,

$y_i=E_i+e_{yi}$,

where e_{xi} and e_{yi} are random variables. It is assumed that e_{xi}s are i.i.d. with $e_{xi}\sim N(0, \sigma^2)$ and that e_{yi}s are i.i.d. with $e_{yi}\sim N(0, k\sigma^2)$. The maximum likelihood estimators are then derived as follows:[8]

$$\hat{\beta} = \frac{S_{yy}-kS_{xx}+\sqrt{\left(S_{yy}-kS_{xx}\right)^2+k\left(2S_{xy}\right)^2}}{2S_{xy}}, \quad (7\text{-}1)$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}, \quad (7\text{-}2)$$

$$\hat{h}_i = \frac{kx_i+\hat{\beta}(y_i-\hat{\alpha})}{k+\hat{\beta}^2}, \quad (7\text{-}3)$$

$$\hat{E}_i = \hat{\alpha} + \hat{\beta}\hat{h}_i, \quad (7\text{-}4)$$

where $S_{xx} = [1/(n-1)]\sum_{i=1}^{n}(x_i-\bar{x})^2$,

$S_{xy} = [1/(n-1)]\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$,

$S_{yy} = [1/(n-1)]\sum_{i=1}^{n}(y_i-\bar{y})^2$.

Equation (7-1) for estimator $\hat{\beta}$ includes $k(=var(e_{yi})/var(e_{xi}))$. The true value of k is unknown. If we assume that k is equal to the ratio of the sample variances ($k=S_{yy}/S_{xx}$), then it reduces to the following:

$$\hat{\beta} = \mathrm{sign}(S_{xy})\sqrt{S_{yy}/S_{xx}}\ . \quad (8)$$

This is known as the GMR estimator.[9]

Hereafter, in the estimation, we use the GMR from the perspective of R^2 (i.e., the coefficient of determination). In the errors-in-variables regression, if the same definition of R^2 as in OLS is applied, it is defined as $R_h^2 = \frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2}$ and $R_E^2 = \frac{\Sigma(\hat{E}_i-\bar{y})^2}{\Sigma(y_i-\bar{y})^2}$ (i=1…n). In

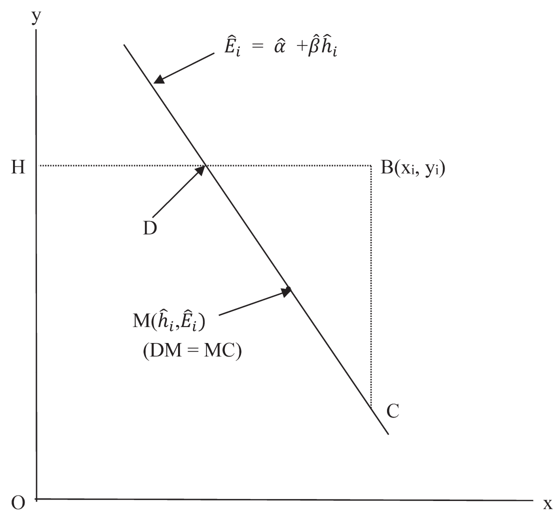

Fig. 5 Observation $B(x_i, y_i)$ and its estimate $M(\hat{t}_i, \hat{E}_i)$ in GMR

Table 1 Estimates of WH contract curve by GMR and OLS regression
(14 manufacturing industries, male, college graduates, age 50–54, 2015)

	(1) GMR	(2) OLS y on x	(3) OLS x on y	(4) OLS x on y/x
(i) Expression	E=α+βh	E=α+βh	h=α+βE	h=α+β(E/h)
(ii) $\hat{\alpha}$ (Standard error)	3927.8 (400.0)	3245.8 (316.1)	199.2 (3.521)	197.0 (2.703)
(iii) $\hat{\beta}$ (Standard error)	−19.02 (2.330)	−15.04 (1.840)	−0.0416 ($5.087*10^{-3}$)	−6.473 (0.651)
(iv) r_{xy} (Correlation coefficient)	−0.791	−0.791	−0.791	−0.791
(v) R^2 (Coefficient of determination)	0.895	0.626	0.626	0.712
(vi) SE of estimate h	3.28		6.20	5.44
SE of estimate E	62.4	118.0		
(vii) Mean of sample h	171.5	171.5	171.5	171.5
Mean of sample E	666.1	666.1	666.1	666.1
(viii) Wage elasticity η (Region of standard error)	−0.170 (−0.189～−0.154)	−0.205 (−0.227～−0.187)	−0.139 (−0.153～−0.124)	−0.147 (−0.161～−0.132)
(ix) Sample number	42	42	42	42

Note:
(a) Row (iv) is the correlation coefficient r_{xy} between x and y.
(b) Row (vi) represents the standard error (SE) of estimates h and E.
(c) Row (vii) denotes the sample mean of x and y.
(d) Row (viii) is the wage elasticity η, which is calculated by w/($\hat{\beta}$–w) (Columns 1 and 2), by w$\hat{\beta}$/(1–w$\hat{\beta}$) (Column 3), and by w$\hat{\beta}$/x (Column 4). They are evaluated at the means of x and y, and w=y/x.
Data source: Basic Survey in Wage Structure (Ministry of Health, Labour and Welfare of Japan).

GMR, using k=S_{yy}/S_{xx}, we obtain $R_h^2= R_E^2$. As the two variables (h and E) are treated symmetrically in the model, it is desirable that $R_h^2= R_E^2$.[10]

Fig. 5 presents the relationship between the observed value B(x_i, y_i) and its corresponding estimate M($\hat{h}_i$, $\hat{E}_i$) in the GMR. Let us then draw vertical and horizontal lines from B. We then let the intersections with the regression line be C(x_i, $\hat{\alpha}$+$\hat{\beta}x_i$) and D((y_i–$\hat{\alpha}$)/$\hat{\beta}$, y_i), respectively. The point of estimate M($\hat{h}_i$, $\hat{E}_i$) is the midpoint of CD.[11]

Table 1 compares GMR with OLS regression using the data presented in Fig. 4. There is a simple relationship between GMR and OLS regression. Column (1) presents the GMR estimate. Column (2) presents the OLS regression estimate of y on x, and Column (3) shows the OLS regression estimate of x on y. $\hat{\beta}$ of the GMR is the geometric mean of the two corresponding OLS estimates, or $(-19.02)^2$=(−15.04)/(−0.0416). The regression line lies between the two OLS regression lines intersecting at the means of x and y.[12]

Rows (iv) and (v) of Table 1 present the correlation coefficient (r_{xy}) and the coefficient of determination (R^2), respectively. As previously stated, for the GMR, $R_h^2= R_E^2$. Furthermore, there is a simple relationship: $R_h^2= R_E^2$ = (1/2)(1+|r_{xy}|).[13]

Row (vi) reports standard errors (SE) of estimates $\hat{h}_i$ and $\hat{E}_i$; in GMR, these are 3.28 and 62.4, respectively. In the OLS regression x on y, the SE of $\hat{h}_i$ is 6.20 (Column 3). Similarly, in the OLS regression y on x, the SE of $\hat{E}_i$ is 118.0 (Column 2). The SEs of the GMR are smaller than those of the OLS regression. Additionally, the ratio is 0.528 (=3.28/6.20=62.4/118.0), which is identical for both $\hat{h}_i$ and $\hat{E}_i$.[14]

Row (viii) presents the wage elasticity of working hours (η), evaluated at the sample means of x and y. In the GMR, the wage elasticity is −0.170. The two wage elasticities by OLS regression (−0.205 and −0.139) are outside the standard error region of the GMR (−0.189 ~ −0.154). In addition, Column (4) presents the OLS regression estimate of x on y/x. It is known that using a wage rate variable measured by y/x results in

Table 2 Estimates of WH contract curve by employee age
(Male, college graduates, 14 manufacturing industries, 2015)

Age	(1) 30–34	(2) 35–39	(3) 40–44	(4) 45–49	(5) 50–54	(6) 55–59
(i) $\hat{\alpha}$ (Standard error)	2078.2 (1148.5)	2533.9 (509.1)	3249.3 (406.1)	3333.7 (357.9)	3927.8 (400.0)	4041.4 (430.1)
(ii) $\hat{\beta}$ (Standard error)	−8.92 (6.197)	−11.06 (2.762)	−15.23 (2.294)	−15.63 (2.059)	−19.02 (2.330)	−19.79 (2.529)
(iii) r_{xy} (Correlation coefficient)	−0.223	−0.536	−0.725	−0.769	−0.791	−0.778
(iv) R^2 (Coefficient of determination)	0.612	0.768	0.862	0.884	0.895	0.889
(v) SE of estimate h	5.51	4.40	3.20	3.42	3.28	3.07
Mean of sample h	185.3	184.3	176.9	173.7	171.5	169.9
(vi) SE of estimate E	49.2	48.7	48.7	53.4	62.4	60.9
Mean of sample E	425.8	496.4	555.0	619.0	666.1	678.6
(vii) Wage elasticity η (Region of standard error)	−0.205 (−0.458～ −0.132)	−0.196 (−0.245～ −0.163)	−0.171 (−0.195～ −0.152)	−0.186 (−0.208～ −0.168)	−0.170 (−0.189～ −0.154)	−0.168 (−0.188～ −0.152)
(viii) Wage elasticity (Log-linear and GMR) (R^2)	−0.202 (0.607)	−0.201 (0.768)	−0.172 (0.865)	−0.184 (0.892)	−0.166 (0.902)	−0.163 (0.891)
(ix) Sample number	42	42	42	42	42	42

Note:
(a) Expression of the estimated equation is $E=\alpha+\beta h$.
(b) Row (v) is the standard error (SE) of the estimates h and the mean of sample h.
(c) Row (vi) is the standard error (SE) of the estimates E and the mean of sample E.
(d) Row (vii) is the wage elasticities η, calculated by $w/(\hat{\beta}-w)$ and evaluated at the means of x and y, and $w=y/x$. In parentheses is the region of the standard error.
(e) Row (viii) is the wage elasticity using the log-linear equation and GMR. The estimated equation is $\ln(E)=a+b\ln(h)$, and the wage elasticity is calculated as $1/(b-1)$.
Data source: Basic Survey in Wage Structure (Ministry of Health, Labour and Welfare of Japan).

division bias. The wage elasticity of −0.147 (Column 4) lies outside the standard error region of GMR. These results indicate that we should not use OLS regression if the explanatory variables are measured with errors.[15]

3.4. GMR estimation results

3.4.1. Estimation results by age group

Table 2 presents the GMR estimates based on employee age. The sample comprises the same group as above (14 manufacturing industries, males, and college graduates).

Row (iv) presents the R^2. For employees aged 50–54 years, it reaches its maximum. For employees over the age of 40, $R^2 > 0.85$. In contrast, for those younger than 40, $R^2 < 0.80$. This is because the wage rate differences in the younger age groups are small. Therefore, the 50–54 age group is the best group to estimate the WH contract curve.

Row (vii) presents the wage rate elasticity of working hours. After the age of 40, the wage elasticity is stable between −0.17 and −0.19. Row (viii) shows the wage elasticity estimated using the log-linear equation and the GMR. Comparing rows (vii) and (viii), we observe that these two estimations yield almost identical results.[16]

3.4.2 Estimation results regarding four industry groups

We apply the same GMR analysis to the other three industry groups: transportation (eight industries), wholesale and retail (12 industries), and hotels et al. (six industries). We assumed that the industries in each group have similar production functions and that the workers in each group have similar preferences.[17]

Table 3 presents the estimates for the four industry groups for 2015. The R^2 is at a passable level. In the groups of six, they are >8.0, and in the other

Table 3 Estimates of the WH contract curve of four industry groups (male, age 50–54, 2015)

	College Graduates				High School Graduates			
	Manu-facturing	Trans-portation	Wholesale and retail	Hotels et al.	Manu-facturing	Trans-portation	Wholesale and retail	Hotels et al.
(i) $\hat{\alpha}$ (Standard error)	3927.8 (400.0)	3000.0 (511.3)	4865.1 (934.8)	2337.1 (359.4)	3114.8 (515.1)	2147.9 (395.2)	2424.1 (455.6)	1683.8 (505.5)
(ii) $\hat{\beta}$ (Standard error)	−19.02 (2.330)	−13.68 (2.908)	−24.84 (5.462)	−10.34 (1.996)	−14.39 (2.844)	−8.79 (2.097)	−10.94 (2.545)	−6.99 (2.713)
(iii) r_{xy} (Correlation coefficient)	−0.791	−0.710	−0.616	−0.793	−0.626	−0.686	−0.595	−0.545
(iv) R^2 (Coefficient of determination)	0.895	0.855	0.808	0.896	0.813	0.843	0.797	0.772
(v) SE of estimate h	3.28	6.98	3.80	4.06	3.48	7.86	4.02	4.17
Mean of sample h	171.5	175.3	171.1	179.8	181.0	187.9	178.9	186.3
(vi) SE of estimate E	62.4	95.5	94.3	42.0	50.0	69.1	44.0	29.1
Mean of sample E	666.1	601.0	616.3	476.8	510.3	495.3	466.7	381.9
(vii) Wage Elasticity η (Region of standard error)	−0.170 (−0.189〜 −0.154)	−0.200 (−0.241〜 −0.171)	−0.127 (−0.157〜 −0.106)	−0.204 (−0.241〜 −0.173)	−0.164 (−0.196〜 −0.141)	−0.231 (−0.282〜 −0.195)	−0.193 (−0.237〜 −0.162)	−0.227 (−0.324〜 −0.174)
(viii) Sample Number	42	24	36	18	42	22	6	18

Note:
(a) Expression of the estimated equation is $E=\alpha+\beta h$.
(b) Row (v) is the standard error (SE) of estimate h and mean of sample h.
(c) Row (vi) is the standard error (SE) of estimate E and mean of sample E.
(d) Row (vii) is wage elasticity η, calculated by $w/(\hat{\beta}-w)$ and evaluated at the means of x and y, and $w=y/x$. In parentheses is the region of the standard error.
Data source: Basic Survey in Wage Structure (Ministry of Health, Labour and Welfare of Japan).

two groups, they are >0.75.

The wage elasticities lie in the range of −0.13 and −0.23. The differences among the industry groups are not large; however, the transportation group has a slightly higher elasticity. Additionally, high school graduates have a slightly higher elasticity than college graduates.

4. Concluding remarks

The constructed model demonstrates that working hours (h) and wage earnings (E) are determined jointly at the equilibrium point on a WH contract curve where demand and supply of workers are equal. In addition, the WH contract curve passes through the intersection of the demand and supply curves of working hours. These results imply that it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours. In addition, it implies that the WH contract curve should be estimated to measure the wage elasticity of working hours.

For the estimation of WH contract curves, GMR was selected for three reasons. First, the WH contract curve equation is a type of measurement error model. That is, both variables (h and E) are measured with errors. Second, the GMR estimator is the maximum likelihood estimator. Third, the coefficients of determination (R^2) of both variables (h and E) are equal in GMR.

In GMR, there is a simple relation between the coefficient of determination (R^2) and correlation coefficient (r_{xy}) as follows: $R^2=(1+|r_{xy}|)/2$.

The estimated wage elasticity of working hours for four industry groups is between −0.13 and −0.23. The transportation group has a slightly higher elasticity than the others, and high school graduates have a slightly higher elasticity than college graduates.

Notes

1. There are many survey articles on the supply

curve of working hours. For examples, See Heckman and Macurdy (1980), Killingsworth (1983), Pencavel (1986), Keane (2011) and Bargain and Peichl (2013).

2. In the model, we aim to synthesize Lewis (1969), Rosen (1974), and Pencavel (2016).
3. An isoprofit curve is derived from the production function AF(L, h) as follows: If the output price is unity, then the firm's profit is π(L, h)=AF(L, h) − L{E(h)+C}, where C denotes fixed costs per employee. As $\pi_L=\pi_h=0$ on an isoprofit curve, we have the following two equations: $\pi_L=AF_L(L, h)-\{E(h)+C\}=0$ and $\pi_h=AF_h(L, h)-LdE(h)/dh=0$. Combining these two equations, we obtain {dE(h)/dh}/{E(h)+C}=(1/L){F_h(L, h)/F_L(L, h)}. The solution E=E(h) to this differential equation is the isoprofit curve. For example, when F(L, h)=$L^{\alpha}h^{\beta}$, the isoprofit curve equation is E+C=$kh^{(\beta/\alpha)}$. Here, k is the integral constant representing the profit level. For more details, refer to Kinoshita (Kinoshita, 1987, p.1274).
4. The WH contract curve is the locus of the tangency points of the isoprofit and indifference curves. Using Equations (1) and (3), we obtain the following Lagrangian equation: $\Gamma(E, h, \lambda)=E-\alpha(h+\beta)^2-\lambda\{h-\gamma(E+\delta)^2-k\}$, where λ denotes the Lagrangian multiplier. From the first-order condition, we obtain the following two equations: $\Gamma_E=1+2\lambda\gamma(E+\delta)=0$ and $\Gamma_h=-2\alpha(h+\beta)-\lambda=0$. By eliminating λ from the equations, we obtain Equation (5).
5. Lewis (Lewis, 1969) describes the following market equilibrium of an industry: All workers have the same quality but different utility functions. All firms have different production functions. Then, the market equilibrium is not a point but a curve with a positive slope. He calls this the "market equalizing wage curve." However, the curve has little information on the supply or demand function for working hours. For more detail, refer to Lewis (1969) and Kinoshita (1987, p.1274).
6. These 14 industries are heavy industries, from the chemical (E16) to the car industry (E31), except for the petroleum (E17) and leather (E20) industries (their respective code numbers are given in parentheses). As each industry consists of three parts based on the number of employees, the total sample number is 42.
7. The Deming regression is an errors-in-variables regression, named after Edwards Deming. The concept of the model was originally introduced in the late 1870s by Adcock and Kummel. This was revived by Koopmans (1937) and later propagated by Deming (1943). For more details, refer to Fuller (1987).
8. The errors-in-variables regression (Deming regression) estimator is a maximum likelihood estimator. On this point, refer to Fuller (1987, p.31), Jensen (2007), and Gillard (2010). The expression E= $\alpha+\beta$h does not imply that E is a dependent variable. Variables h and E are both symmetric in the model. On this point, see Fuller (1987, p.30).
9. (i) Equation (7-1) includes cases of the OLS regression estimator. Solving (7-1) with respect to k, we have $k=\hat{\beta}(\hat{\beta}S_{xy}-S_{yy})/(S_{xy}-\hat{\beta}S_{xx})$. Therefore, if k → ∞ (i.e., var(e_{xi})→0), $\hat{\beta}\to S_{xy}/S_{xx}$, which is equal to the $\hat{\beta}$ of the OLS regression estimate of y on x. If k=0 (i.e., var(e_{yi})=0), $\hat{\beta}=S_{yy}/S_{xy}$. This is the inverse of the OLS regression estimate of x on y. Additionally, $d\hat{\beta}/dk>0$ is derived from the above equation.
(ii) $\{S_{yy}/S_{xx}\}=\{S_{xy}/S_{xx}\}\{S_{yy}/S_{xy}\}$, which shows that Equation (8) is the geometric mean of the two corresponding OLS estimates.
10. (i) From (7-2) and (7-4), we have $\hat{\beta}^2=\frac{\Sigma(\hat{E}_i-\bar{y})^2}{\Sigma(\hat{h}_i-\bar{\bar{x}})^2}$. In the GMR, from Equation (8), we have $\hat{\beta}^2=\frac{\Sigma(y_i-\bar{y})^2}{\Sigma(x_i-\bar{x})^2}$. Therefore, $R_E^2=\frac{\Sigma(\hat{E}_i-\bar{y})^2}{\Sigma(y_i-\bar{y})^2}=\hat{\beta}^2\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(y_i-\bar{y})^2}=\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2}=R_h^2$. (ii) Another definition of R_h^2 is $R_h^2=1-\frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}$. The equality $1-\frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}=\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2}$ is proved using the normal equation. Refer to Jensen (2007) for the normal equation in the Deming regression. (iii) It is shown that $dR_t^2/dk>0$ and $dR_E^2/dk<0$. In addition, if k=0, $R_E^2=1.0$, and, if k →∞, $R_h^2\to 1.0$.
11. (i) In GMR, k=$\hat{\beta}^2$. Inserting this into (7-3), we have $\hat{h}_i=(1/2)[x_i+(1/\hat{\beta})(y_i-\hat{\alpha})]$=(1/2)(BH+DH). (ii)Barker et al. (1988) presented the 'least triangular approach' to obtain a GMR estimator. This minimizes the sum of the triangular area BCD of all observed points.
12. To calculate the standard errors of $\hat{\alpha}$ and $\hat{\beta}$ in GMR (Column 1), refer to Fuller (1987, pp.30-36).
13. From note (11), $\hat{h}_i=(yi+\hat{\beta}x_i-\hat{\alpha})/2\hat{\beta}$; we insert this into the equation below:

$R_h^2 = \frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2} = \frac{\Sigma\{(y_i+\hat{\beta}x_i-\hat{\alpha}-2\hat{\beta}\bar{x})/2\hat{\beta}\}^2}{\Sigma(x_i-\bar{x})^2}$

$= \frac{1}{(2\hat{\beta})^2}\frac{\Sigma\{(y_i-\bar{y})+\hat{\beta}(x_i-\bar{x})\}^2}{\Sigma(x_i-\bar{x})^2}$

$= 1/2 + \frac{1}{2\hat{\beta}}\frac{\Sigma(y_i-\bar{y})(x_i-\bar{x})}{\Sigma(x_i-\bar{x})^2}$

$= (1/2)(1+|r_{xy}|)$

As $1 \geqq |r_{xy}| \geqq 0$, the region of the R^2 is $1.0 \geqq R^2 \geqq 0.5$ in GMR.

14. In GMR, the estimates $\hat{h}_i$ and $\hat{E}_i$ are given by $\hat{h}_i=(y_i+\hat{\beta}x_i-\hat{\alpha})/(2\hat{\beta})$ and $\hat{E}_i=(y_i+\hat{\beta}x_i+\hat{\alpha})/2$. Thus, the errors are $x_i-\hat{h}_i=(-y_i+\hat{\beta}x_i+\hat{\alpha})/(2\hat{\beta})$, and $y_i-\hat{E}_i=(y_i-\hat{\beta}x_i-\hat{\alpha})/2$, respectively. The standard errors of $\hat{h}_i$ and $\hat{E}_i$ are then calculated as follows: $\{\Sigma(x_i-\hat{h}_i)^2/(n-2)\}^{(1/2)}$ and $\{\Sigma(y_i-\hat{E}_i)^2/(n-2)\}^{(1/2)}$, where n is the sample number.
15. On division bias, refer to Borjas (1980).
16. When the log-linear equation ln(E)=a+bln(h) is assumed, this implies a constant wage elasticity of working hours. The wage elasticity was then calculated as 1/(b–1).
17. The code numbers of the industries are as follows: transportation: (H42–H49), wholesale and retail: (I50–I61), and hotels et al.: (M75–N80).

Mathematical note

1. Concerning note (8), (i)

Proof of $d\hat{\beta}/dk>0$

$k = \hat{\beta}(\hat{\beta}S_{xy} - S_{yy})/(S_{xy} - \hat{\beta}S_{xx})$

$dk/d\hat{\beta} = \{1/(S_{xy} - \hat{\beta}S_{xx})^2\}\{(S_{xy})(-\hat{\beta}^2S_{xx}+2\hat{\beta}S_{xy}-S_{yy})\} > 0$

(+)　(−)　(−)

Third term is negative, because $S_{xy}^2-S_{xx}S_{yy}<0$. Q.E.D.

2. Concerning note (9), (ii)

Proof of $\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2} = 1 - \frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}$

(i) Normal equations in Deming regression are as follows: (Jensen 2007)

(L is likelihood function)

① $\delta L/\delta\hat{\beta} = \Sigma(y_i-\hat{\alpha}-\hat{\beta}\hat{h}_i)\hat{h}_i = 0$　(eq. #1)

② $\delta L/\delta\hat{\alpha} = \Sigma(y_i-\hat{\alpha}-\hat{\beta}\hat{h}_i) = 0$　(eq. #2)

(ii) From normal equations ① and ②, the following two equations are derived:

$\Sigma(y_i-\hat{\alpha}-\hat{\beta}x_i)\hat{h}_i = 0$　(eq. #3)

$\Sigma(y_i-\hat{\alpha}-\hat{\beta}x_i) = 0$　(eq. #4)

Proof:

Using equation (7-3)

$(y_i-\hat{\alpha}-\hat{\beta}\hat{h}_i) = (y_i-\hat{\alpha}) - \hat{\beta}\{\frac{kx_i+\hat{\beta}(y_i-\hat{\alpha})}{k+\hat{\beta}^2}\}$

$= \frac{k}{k+\hat{\beta}^2}(y_i-\hat{\alpha}-\hat{\beta}x_i)$　(eq. #5)

Inserting eq. #5 into normal equation ① and ②, eq. #3 and eq. #4 are derived.

(iii) Proof of $\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2} = 1 - \frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}$

$\frac{\Sigma(\hat{h}_i-\bar{x})^2}{\Sigma(x_i-\bar{x})^2} = 1 - \frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}$

$\Updownarrow$

$\sum(\hat{h}_i-\bar{x})^2 = \sum(x_i-\bar{x})^2 - \Sigma(\hat{h}_i-x_i)^2$

$= \sum(\hat{h}_i-\bar{x})(2x_i-\bar{x}-\hat{h}_i)$

$\Updownarrow$

$2\sum(\hat{h}_i-\bar{x})(\hat{h}_i-x_i) = 0$

$\Updownarrow$

$\sum(\hat{h}_i-x_i)\hat{h}_i - \bar{x}\sum(\hat{h}_i-x_i) = 0$

(i)　(ii)

We show that both terms (i) and (ii) are 0.

Proof of the first term (i)= 0 :

Using equation (7-3)

$\sum(\hat{h}_i-x_i)\hat{h}_i = \Sigma\{\frac{kx_i+\hat{\beta}(y_i-\hat{\alpha})}{k+\hat{\beta}^2} - x_i\}\hat{h}_i$

$= \{\frac{\hat{\beta}}{k+\hat{\beta}^2}\}\Sigma(y_i-\hat{\alpha}-\hat{\beta}x_i)\hat{h}_i = 0$

(from eq. #3)

Proof of the second term (ii) = 0 ;

Using equation (7-3)

$\sum(\hat{h}_i-x_i) = \Sigma\{\frac{kx_i+\hat{\beta}(y_i-\hat{\alpha})}{k+\hat{\beta}^2} - x_i\}$

$= \frac{\hat{\beta}}{k+\hat{\beta}^2}\Sigma(y_i-\hat{\alpha}-\hat{\beta}x_i) = 0.$

(from eq. #4)　Q.E.D.

3. Concerning note 9, (iii),

Proof of $dR_h^2/dk>0$ and $dR_E^2/dk<0$

We show $dR_h^2/dk>0$.

As $R_h^2 = 1 - \frac{\Sigma(\hat{h}_i-x_i)^2}{\Sigma(x_i-\bar{x})^2}$, it is equivalent to show

$d/dk\{\Sigma(\hat{h}_i-x_i)^2\}<0.$

Using equation (7-3)

$$\Sigma\left(\hat{h}_i - x_i\right)^2 = \Sigma\left\{\frac{kx_i + \hat{\beta}(y_i - \hat{\alpha})}{k + \hat{\beta}^2} - x_i\right\}^2$$

$$= \left(\frac{\hat{\beta}}{k + \hat{\beta}^2}\right)^2 \Sigma\left(y_i - \hat{\alpha} - \hat{\beta}x_i\right)^2 \qquad \text{(eq. \#6)}$$

Using equation (7-4)

$$\Sigma\left(\hat{E}_i - y_i\right)^2 = \Sigma\left(\hat{\alpha} + \hat{\beta}\hat{h}_i - y_i\right)^2$$

$$= \Sigma\left[\hat{\alpha} - y_i + \hat{\beta}\frac{1}{k + \hat{\beta}^2}\left\{kx_i + \hat{\beta}(y_i - \hat{\alpha})\right\}\right]^2$$

$$= \left(\frac{-k}{k + \hat{\beta}^2}\right)^2 \Sigma\left(y_i - \hat{\alpha} - \hat{\beta}x_i\right)^2$$

(eq. #7)

Dividing eq. #7 by eq. #6:

$$\Sigma\left(\hat{E}_i - y_i\right)^2 / \Sigma\left(\hat{h}_i - x_i\right)^2 = k^2 / \hat{\beta}^2 \qquad \text{(eq. \#8)}$$

From Jensen (2007),

$$\sigma^2 = \left(\frac{1}{2kn}\right)\left[k\Sigma\left(x_i - \hat{h}_i\right)^2 + \Sigma\left(y_i - \hat{E}_i\right)^2\right] \qquad \text{(eq.\#9)}$$

$$= \left(\frac{1}{2n}\right)\left[\Sigma\left(x_i - \hat{h}_i\right)^2 + \frac{k}{\hat{\beta}^2}\Sigma\left(\hat{h}_i - x_i\right)^2\right]$$

$$\therefore \quad \Sigma\left(x_i - \hat{h}_i\right)^2 = 2n\sigma^2\left\{\frac{\hat{\beta}^2}{(\hat{\beta}^2 + k)}\right\} \qquad \text{(eq.\#10)}$$

$$d/dk\left\{\Sigma\left(x_i - \hat{h}_i\right)^2\right\} = 2n\sigma^2 \times 1/\left(\hat{\beta}^2 + k\right)^2\left\{2\hat{\beta}\frac{d\hat{\beta}}{dk}k - \hat{\beta}^2\right\} < 0$$

The third term $\{2\hat{\beta}\frac{d\hat{\beta}}{dk}k - \hat{\beta}^2\}$ is negative, because $\hat{\beta} < 0$, $\frac{d\hat{\beta}}{dk} > 0$ and $k > 0$. Q.E.D.

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