The Bank of Japan's yield curve control: A model-based evaluation

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Abstract

The Bank of Japan has adopted the yield curve control (YCC) monetary policy since September 2016. While traditional monetary policies control the short-term rates, YCC tries to control and hold the 10-year Japanese government bond (JGB) yields at around zero percent. Moreover, YCC is implemented by purchases of the JGBs over any maturities. This study evaluates the effects of YCC by modifying a stochastic model proposed by Jarrow and Li (2014) [Review of Derivatives Research 17 (3), pp. 287-321]. Their model extends the Heath-Jarrow-Morton model to incorporate the effects of government bond purchases of a central bank. The model parameters are estimated using a Kalman filter technique. The empirical results reveal that YCC lowers the yield curve by 1.7553% per year.

Keywords : yield curve control; the Bank of Japan; Japanese government bond purchase; Jarrow-Li model; Kalman filter

JEL classifications : E43; E52; E58; G12

1 Introduction

Central banks have adopted several new monetary policies since the late 1990s. This study examines the case of yield curve control (YCC) of the Bank of Japan (BOJ). A brief history of the monetary policies of the BOJ is summarized as follows. The BOJ changed its money market operation guidelines on 9 September 1998.[1] Since then, the BOJ has controlled uncollateralized overnight call rate to move on average around the target rate. The BOJ implemented a zero-interest rate policy from 12 February 1999 to 11 August 2000.[2] On 19 March 2001, the BOJ initiated a quantitative easing policy, changing the main operating target for money market operations from the uncollateralized overnight call rate to the outstanding balance of current accounts held with the BOJ.[3] On 5 October 2010, the BOJ introduced a comprehensive monetary easing policy to ensure that the uncollateralized overnight call rate remained between 0% and 0.1%.[4] On 4 April 2013, the BOJ proposed a quantitative and qualitative monetary easing to change the main operating target for money market operations from the uncollateralized overnight call rate to the monetary base.[5]

Drastic changes occurred in 2016. On 29 January 2016, the BOJ introduced a quantitative and qualitative monetary easing with a negative interest rate.[6] The BOJ pursued monetary easing making full use of the possible measures in terms of three dimensions, quantity, quality, and interest rate. First, the BOJ applied a negative interest rate of minus 0.1% to the current accounts of financial institutions. Second, the BOJ conducted money market operations to increase the monetary base by about 80 trillion yen annually. Third, the BOJ purchased Japanese government bonds (JGBs) to increase the outstanding amount by about 80

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1 https://www.boj.or.jp/en/announcements/release_1998/k980909c.htm/.

2 https://www.boj.or.jp/en/announcements/release_1999/k990212c.htm/, https://www.boj.or.jp/en/announcements/release_2000/k000811.htm/.

3 https://www.boj.or.jp/en/announcements/release_2001/k010319a.htm/.

4 https://www.boj.or.jp/en/announcements/release_2010/k101005.pdf.

5 https://www.boj.or.jp/en/announcements/release_2013/k130404a.pdf.

6 https://www.boj.or.jp/en/announcements/release_2016/k160129a.pdf.

trillion yen annually. In an attempt to reduce the interest rates over the entire yield curve, the BOJ purchased JGBs flexibly based on the financial market conditions. The BOJ introduced yield curve control (YCC) to implement quantitative and qualitative monetary easing on 21 September 2016.[7] As regards the short-term policy interest rate, the BOJ applied a negative interest rate of minus 0.1% to the policy-rate balances in the current accounts of financial institutions. As for the long-term interest rates, the BOJ purchased JGBs in order to hold the 10-year JGB yields at around 0%. The BOJ also introduced new modes of market operation to facilitate YCC, such as outright purchases of JGBs with yields designated by the BOJ (fixed-rate purchase operations) and fixed-rate funds-supply operations for up to 10 years (extending the longest maturity of operation from one year). As of June 2022, YCC is under way.

As described above, the BOJ introduced several monetary policies. Of those, YCC is an ambitious policy because it aims to control the short-term rates as well as the term structure of interest rates, including the long-term rates. Few studies have investigated the effects of YCC. Nakano et al. (2018) investigate the time series of the yields by introducing supply and demand variables. Hattori and Yoshida (2020) show that the effects of YCC are limited to the JGB market. They also show that the time series of the yield changed from stationary to nonstationary and the yield volatility declined after the introduction of YCC. However, no study has quantitatively investigated the effects of YCC on the term structure of interest rates.

The BOJ has performed YCC through the outright purchase of JGBs. However, naïve investigations of the relationship between change in spot rate and amount of BOJ's JGB purchases are meaningless. BOJ's JGB purchase has one of three effects: it (1) lowers the interest rate, (2) prevents the interest rate from rising, or (3) reduces the interest rate increment. Moreover, what would happen to the interest rates if the BOJ did not purchase JGBs is not clear. Therefore, we need a new quantitative model to explain the interest rate dynamics both in the absence and under YCC.

Jarrow and Li (2014) propose an interest rate model that would explicitly explain the quantitative impact of the Fed's trade on the Treasury market. They consider two types of interest rates, the one observed in the government bond market under central bank operations (I call this the controlled interest rate), and the other that would be observed if the central bank operations were absent (I call this the uncontrolled interest rate). The uncontrolled interest rate is formulated under the Heath-Jarrow-Morton (1992) framework. They then construct a controlled interest rate that deviates from the uncontrolled interest rate but reverts to the uncontrolled rate. They applied the model to evaluate the effect of the Fed's quantitative easing policy.

This study aims to quantitatively investigate the effects of YCC on the term structure of spot rates estimated from JGB prices. I evaluate the amplitude of decline in term structure of spot rates due to YCC. This study modifies Jarrow and Li's (2014) model to consider not the termination, but initiation of the central bank's bond purchase program. This empirical study of the JGB market shows that YCC lowers the term structure of spot rates by 1.7553% per year. By product, the empirical results illustrate an applicability of the cubic B-spline functions to estimate the term structure of interest rates even under zero or negative interest rates.

The remainder of this paper is structured as follows. Section 2 analyzes BOJ's JGB purchases. Section 3 explains the model and method used, discussing the estimation of term structure of spot rates and the modification of Jarrow and Li's (2014) model. Section 4 explains the data and presents the empirical results. Section 5 concludes the paper.

2　YCC and JGB purchases by the BOJ

First, I examine the JGBs outstanding balances held with the BOJ by maturity period.[8] The JGBs outstanding balances held with the BOJ are reported at 1-month intervals from June 2001 to April 2014 and

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7 https://www.boj.or.jp/en/announcements/release_2016/k160921a.pdf.

8 The BOJ holds fixed- and floating-rate JGBs, and inflation-indexed JGBs. I concentrate on the fixed-rate JGBs because we are interested in the relation between the yield curve and BOJ's monetary policy.

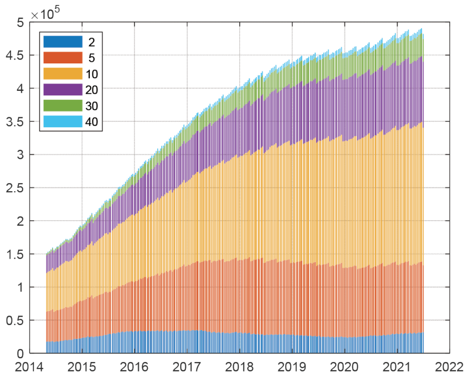

Fig. 1 JGBs outstanding balance held with the BOJ by maturity period (in billion yen). The JGB maturity periods are 2, 5, 10, 20, 30, and 40 years.

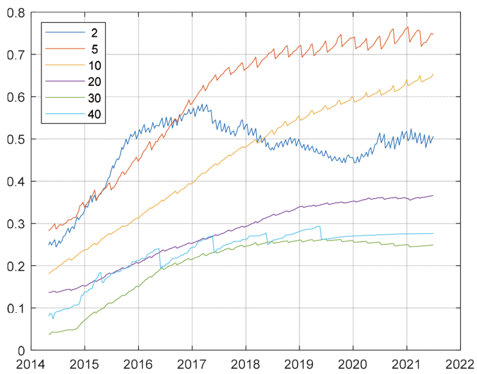

Fig. 2 Ratio of JGBs outstanding balance held with the BOJ to the total outstanding balance by maturity period. The JGB maturity periods are 2, 5, 10, 20, 30, and 40 years.

10-day intervals from May 2014. The JGB maturity periods are 2, 5, 10, 20, 30, and 40 years. Fig. 1 presents a stacked bar chart of the JGBs outstanding balances held with the BOJ by maturity period. The JGBs outstanding balances grow monotonically, especially those with maturity periods of 10, 20, and 30 years. The ratio of JGBs outstanding balance held with the BOJ to the total outstanding balance for each maturity period is plotted in Fig. 2. The ratio shows an increase for all maturity periods except two years. At the end of June 2021, the ratios of JGBs with maturity periods of five and ten years exceeded 70% and 65%, respectively. The ratio plots do not change smoothly but show sudden drops in long-term cycles along with short-term zig-zag fluctuations because new JGB issues temporally decrease the ratio and redemptions of issued JGBs change the ratio.

Next, the JGBs outstanding balance held with the BOJ is examined, not by its maturity period but remaining term to maturity, to measure the magnitude of JGB purchases made under YCC. To do so, I first discretize the remaining terms to maturity from 0 to 20 years in one-year increments and those from 25 to 40 years in five-year increments to construct grid points. I then distribute both the total outstanding balance and JGBs outstanding balance held with the BOJ to the grid points in order to preserve their present values and durations. Subsequently, I calculate the ratio of JGBs outstanding balance with the BOJ to the total outstanding balance by its remaining term to maturity. This ratio is presented as a contour plot in Fig. 3. From this figure, the JGBs with terms to maturity of up to 12 years are increasingly held with

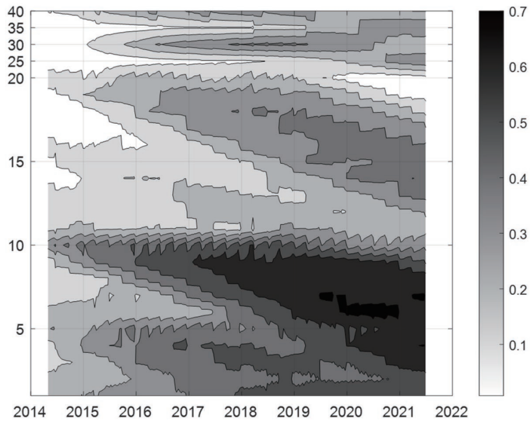

Fig. 3 Ratio of JGBs outstanding balance held with the BOJ to the total outstanding balance by remaining term to maturity. The x-axis represents calendar date and the y-axis represents the remaining term to maturity of the JGBs.

the BOJ after initiation of YCC program, which started on 21 September 2016.

The BOJ uses YCC to lower the 10-year yield to 0%. From Figs. 1, 2, and 3, the BOJ holds JGBs with maturities of up to 40 years. Moreover, the BOJ focuses on purchasing JGBs with maturities longer than 10 years. Thus, I analyse the term structure of spot rates for maturities up to 12 years.

3 Estimation of the effect of BOJ's YCC

3.1 Estimation of term structure of interest rates

YCC can reduce long-term yields through the outright purchase of fixed-rate JGBs of all maturities. This study estimates the yield curve as a term structure of spot rates and evaluates the effect of YCC by examining the term structure of spot rates. Thus, I first explain how the term structure of spot rates is estimated.

The term structure of spot rates is estimated using cubic B-spline functions, as proposed by Fisher et al. (1994). I set the current time at 0 and denote the horizon by T^*. I then introduce knot points $\{s_k\}_{k=1}^{K}$ satisfying $s_k < s_{k+1}$, $s_0 = 0$, and $s_K = T^*$, and define the augmented set of knot points $\{d_k\}_{k=1}^{K+6}$ such that

$$d_1 = d_2 = d_3 = s_1, \tag{3.1}$$

$$d_{k+3} = s_k \quad (k = 1, \ldots, K), \tag{3.2}$$

and

$$d_{K+4} = d_{K+5} = d_{K+6} = s_K. \tag{3.3}$$

The basis of the $(r-1)$-th B-spline functions $\phi_k^r(\tau)$ $(0 \le \tau \le T^*$ and $k = 1, \ldots, K+2)$ is recursively defined as

$$\phi_k^r(\tau) = \frac{\tau - d_k}{d_{k+r-1} - d_k}\phi_{k-1}^r(\tau) + \frac{d_{k+r} - \tau}{d_{k+r} - d_{k+1}}\phi_{k-1}^r(\tau)$$
$$(r = 2, 3, \ldots), \tag{3.4}$$

$$\phi_k^1(\tau) = \begin{cases} 1 & \text{when } d_k \le \tau < d_{k+1} \\ 0 & \text{otherwise} \end{cases}. \tag{3.5}$$

By iterating this procedure, I construct the basis of the cubic B-spline functions, $\phi_k^4(\tau)(k = 1, \ldots, K+2)$.

The time-t spot rate maturing at time T, $R(t, T)$, can be expressed using the cubic B-spline function as

$$R(t,T) = \sum_{k=1}^{K+2} \beta_k \, \phi_k^r(T - t). \tag{3.6}$$

The time-t discount factor maturing at time T, $D(t,T)$, can be written as

$$D(t,T) = \exp\big(-(T - t)\, R(t,T)\big). \tag{3.7}$$

I estimate β_k $(k = 1, \ldots, K+2)$, which minimizes the squared sum of pricing errors for fixed-rate JGBs with maturity periods of 2, 5, 10, and 20 years. In the empirical study, I set knot points at 0 to 12 months with three-month increments, 18 months, and 2 to 20 years with one-year increments. This choice of knot points enables a good approximation of the term structure of spot rates estimated from JGB prices.

3.2 The interest rate dynamics under YCC

In this study, I modify the model proposed by Jarrow and Li (2014) and apply it to the JGB market. Their model considered two types of interest rates: one that is observed in the absence of central bank manipulation and another that is observed under central bank monetary operations. The time-t forward rate maturing at time T without YCC and that with YCC are denoted by $f(t, T)$ and $F(t, T)$, respectively. Likewise, the time-t spot rates maturing at time T without YCC and that with YCC are denoted by $r(t, T)$ and $R(t, T)$, respectively. I call $f(t, T)$ and $r(t, T)$ uncontrolled interest rates, and $F(t, T)$ and $R(t, T)$ controlled interest rates.

I consider the introduction of YCC and evaluate the effects of YCC on the term structure of spot rates. While Jarrow and Li (2014) investigate the effects of the termination of outright purchase of government bonds by the central bank, this study investigates the effects of the initiation of outright purchase by the central bank. The relationship between the two forward rate types can be expressed by the differential equation

$$dF(t,T) = df(t,T) - 1_{\{t \ge \tau\}}\psi(t,T)dt$$
$$- \lambda(t,T)\big(F(t,T) - f(t,T)\big)dt, \tag{3.8}$$

where $\psi(t, T)$ is the time-t impact of YCC on the forward rate $F(t, T)$ maturing at time T, $\lambda(t, T)$ is the speed of controlled forward rate $F(t, T)$ reverting to the uncontrolled forward rate $f(t, T)$, τ is the time of starting YCC, and $1_{\{\cdot\}}$ is an indicator function. This model is interpreted as follows: BOJ's outright purchases of JGB deviate the controlled forward rate $F(t, T)$ from the current level by $\psi(t, T)\,dt$ and the controlled forward rate $F(t, T)$ reverts to the level of the uncontrolled forward rate $f(t, T)$ with speed $\lambda(t, T)$. The differential equation (3.8) can then be solved as follows:

$$F(t,T) = f(t,T)$$
$$- 1_{\{t \ge \tau\}} \int_\tau^t \exp\left(-\int_s^t \lambda(u,T)du\right)\psi(s,T)ds. \tag{3.9}$$

In this study, I assume that the impact of YCC, $\psi(t, T)$, is constant over time for all maturities because of the limitation of data.[9] The impact could be

[9] Jarrow and Li (2014) assume that $\psi(t, T)$ is a function of term to maturity, $T - t$, to obtain a state-space representation, but their mathematical calculation is erroneous. Their calculation would hold if $\psi(t, T)$ were a function dependent only on T.

measured if intra-day tick data were available, but JGBs are traded over the counter, making it impossible to obtain tick-by-tick data. At most, we can observe the JGBs outstanding balance held with the BOJ at 10-day intervals. Likewise, I assume that the reverting speed of the controlled forward rate, $\lambda(t, T)$, is constant. Thus, I have

$$F(t,T) = f(t,T) - 1_{\{t \geq \tau\}} \frac{\psi}{\lambda}\left(1 - e^{-\lambda(t-\tau)}\right). \tag{3.10}$$

I assume that the uncontrolled forward rate at time t maturing at T, $f(t, T)$, follows the Vasicek (1977) model in the Heath, Jarrow and Morton (1992) framework:

$$df(t,T) = \sigma(t,T)\left(\int_t^T \sigma(u,T)du\right)dt + \sigma(t,T)dW_t^{\mathbb{Q}}, \tag{3.11}$$

$$\sigma(t,T) = \sigma_r e^{-\kappa(T-t)}, \tag{3.12}$$

where $dW_t^{\mathbb{Q}}$ is a Wiener process under martingale measure $\mathbb{Q}$. I formulate this model using the short rate,[10] which is defined as

$$r_t \coloneqq r(t) = \lim_{T \to t} f(t,T). \tag{3.13}$$

and is explicitly expressed as

$$r_t = f(0,t) + \frac{{\sigma_r}^2}{2\kappa^2}(1 - e^{-\kappa t})^2 + \int_0^t \sigma_r e^{-\kappa(t-s)} dW_s^{\mathbb{Q}}. \tag{3.14}$$

The differential equation of the short rate is

$$dr_t = \kappa(\theta_t - r_t)dt + \sigma_r dW_t^{\mathbb{Q}}, \tag{3.15}$$

where

$$\theta_t = \frac{1}{\kappa}\left(\frac{\partial}{\partial t} f(0,t)\right) + f(0,t) + \frac{{\sigma_r}^2}{2\kappa^2}(1 - e^{-2\kappa t}). \tag{3.16}$$

In this empirical study, I further assume that the level of mean reversion, θ_t, is constant.

The model parameters, θ, κ, σ_r, λ, and ψ, are estimated using a Kalman filter technique (see Durbin and Koopman, 2012). First, I formulate a state-space model regarding the short rate as state variable. The state equation is written as

$$r(t+\Delta t) = e^{-\kappa \Delta t} r(t) + \theta(1 - e^{-\kappa \Delta t}) + \epsilon_t,$$
$$\epsilon_t \sim N\left(0, \frac{{\sigma_r}^2}{2\kappa^2}(1 - e^{-2\kappa \Delta t})\right). \tag{3.17}$$

The observation equations can be given as

$$R(t,T_i) = \frac{1}{T_i - t} B(t,T_i) r(t) - \frac{1}{T_i - t} A(t,T_i) - 1_{\{t \geq \tau\}} \frac{\psi}{\lambda}\left(1 - e^{-\lambda(t-\tau)}\right) + \epsilon(t,T_i),$$
$$\epsilon(t,T_i) \sim N(0, \sigma_i^2), \tag{3.18}$$

where

$$A(t,T_i) = \left(\theta - \frac{{\sigma_r}^2}{2\kappa^2}\right)\left(B(t,T_i) - (T_i - t)\right) - \frac{{\sigma_r}^2}{4\kappa} B(t,T_i)^2, \tag{3.19}$$

and

$$B(t,T_i) = \frac{1}{\kappa}\left(1 - \exp\left(-\kappa(T_i - t)\right)\right). \tag{3.20}$$

In the estimation, the term to maturity, $T_i - t$, is set from 1 to 12 years at one-year increments.

4 Empirical study

4.1 Estimation of the term structure of spot rates

The JGB price data are obtained from the Japan Securities Dealers Association. The BOJ disclosed their JGB holdings issue-by-issue on a monthly basis until April 2014. From May 2014, the BOJ has disclosed their JGB holdings at 10-day intervals. This study analyses the 10-day interval data from 30 April 2014 to 30 June 2021. The number of observations is 259. I set the time 0 to 30 April 2014 and τ to 21 September 2016, the date YCC was introduced.

Fig. 4 depicts the term structure of spot rates estimated from JGB prices. The term structure of spot rates continued to decline from 30 April 2014, recording a historical low on 8 July 2016. Thereafter, the term structure of spot rates rose slightly. The 10-year spot rates have never exceeded 0.2% since 21 September 2016. Thus, YCC works well to control the long-term interest rates. The quantile of JGB pricing errors is shown in Fig. 5. The pricing errors for 95% of JGB issues are within ±0.1 Japanese yen with a face value of 100 Japanese yen. Thus, I conclude that the

[10] The mathematical manipulation of stochastic calculus is given in detail e.g., Brigo and Mercurio (2006).

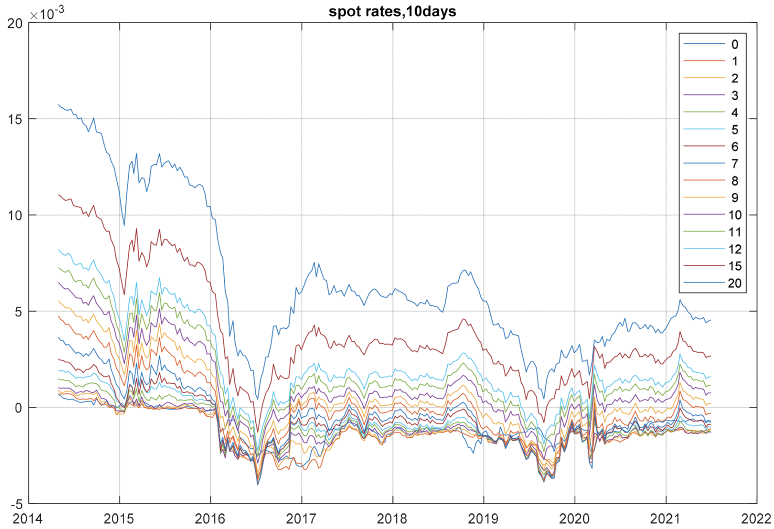

Fig. 4 Term structure of spot rates. Spot rates maturing at 0, 1, ..., 12, 15, and 20 years are depicted.

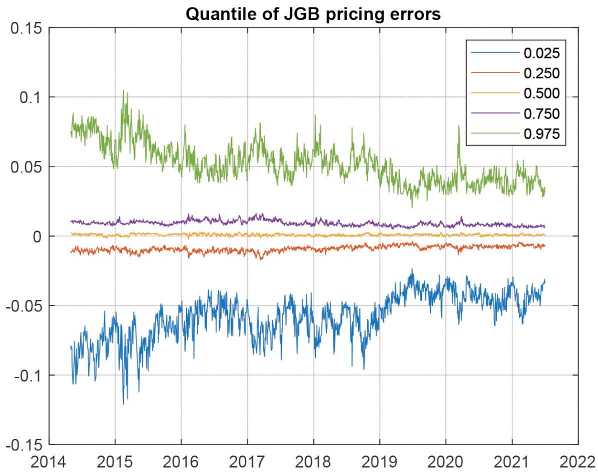

Fig. 5 Quantile of JGB pricing errors measured in Japanese yen. The face value of JGBs is 100 Japanese yen.

cubic B-spline functions are appropriate to estimate the term structure of spot rates even under zero or negative interest rates.

4.2 Preliminary analysis of YCC

I calculate BOJ's JGB purchases from the BOJ's JGB holdings reported at 10-day intervals. BOJ's JGB purchases are plotted by their term to maturity in Fig. 6. The descriptive statistics of BOJ's JGB purchases is presented in Table 1. The BOJ mainly purchased JGBs with remaining term to maturity of 2, 5, and 10 years. For all maturities, BOJ's JGB purchases are highly volatile during the observation period.

One might expect BOJ's JGB purchases to be inversely related to the changes in JGB yield. The correlation coefficients between changes in spot rates and BOJ's JGB purchases are provided in Table 2. The maximum and minimum of the correlation coefficient are 0.132 and −0.104, respectively, showing almost no correlation between changes in spot rates and BOJ's JGB purchases. I conjecture that the data sampling interval of 10-days is too long to detect any changes in spot rates due to BOJ's JGB purchases. As stated above, the BOJ's JGB purchase amount is not an appropriate explanatory variable to measure the effect of YCC. Therefore, I take the indicative variable of whether YCC is effective or not as an explanatory variable.

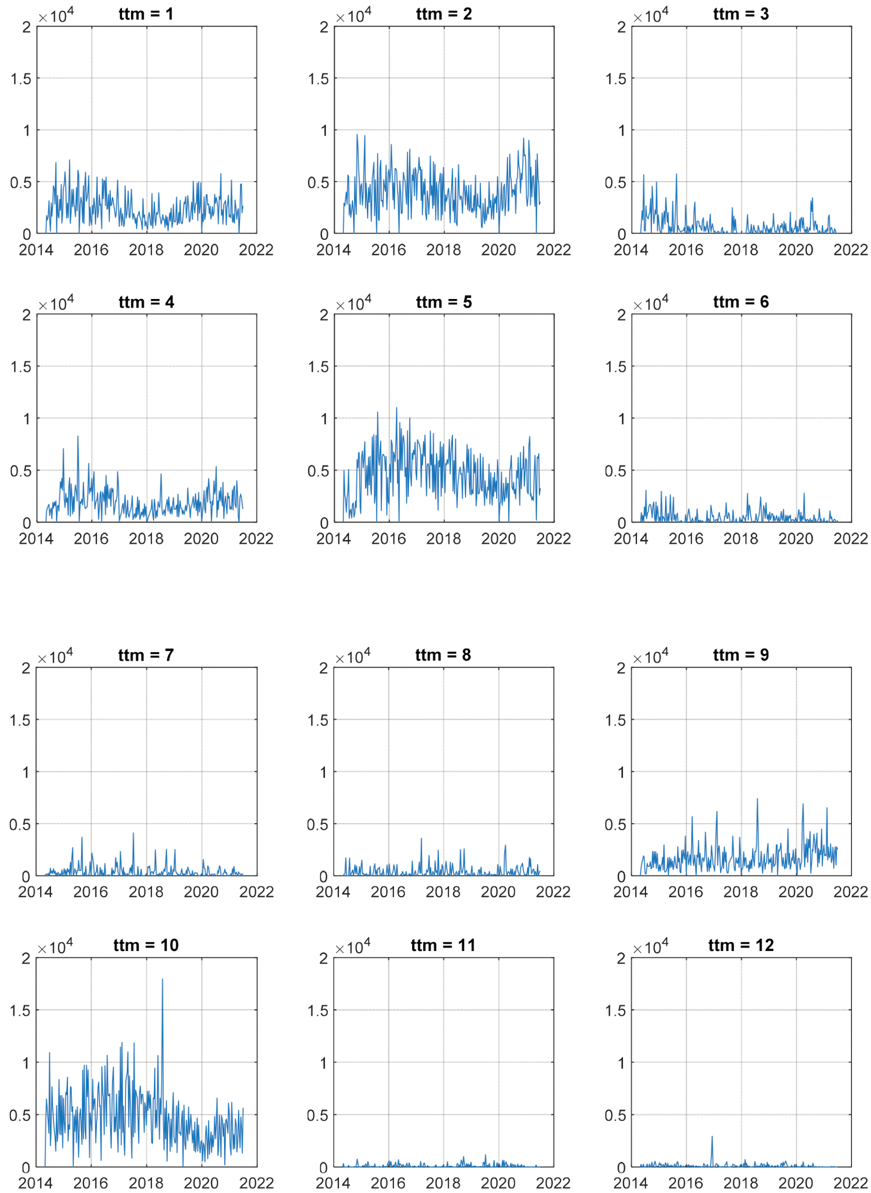

Fig. 6 BOJ's JGB purchases by term to maturity (in billion yen).

4.3 Evaluation of the effects of YCC

I estimate model parameters, θ, κ, σ_r, λ, and ψ, using the Kalman filter technique. For the observation equation, I take the terms to maturity from 1 to 12 years in one-year increments. I apply two models to evaluate the effects of YCC. Model I does not consider the effects of YCC. Model II considers the effects of YCC. Model II can be reduced to Model I by setting $\lambda=0$ and $\psi=0$. Thus, Model I is a restricted version of Model II. The estimates are presented in Table 3. The log-likelihoods of Models I and II are $\mathcal{L}_1=18718.60$ and $\mathcal{L}_2=18928.73$, respectively. The likelihood ratio test, $LR=2(\mathcal{L}_2-\mathcal{L}_1)\sim\chi^2(2)$, shows that Model II is superior to Model I. Thus, the dynamics of interest rates are more appropriately described by the stochastic model including the effect of YCC. Thus, I investigate Model II.

All the coefficients of Model II are statistically significant. The mean reversion level of the controlled forward rate is 4.4231%. This level is high compared to the spot rates in the JGB market. The reverting speed of the controlled forward rate is 0.609534, which is stronger than the mean reversion speed of 0.014042. The impact of YCC is 1.7553% per year. I thus conclude that the YCC effect can be successfully measured and YCC works well to lower the 10-year

Table 1 Descriptive statistics of BOJ's JGB purchases (in billion yen). Here, JGB(n) stands for BOJ's n-year JGB purchase.

	JGB(1)	JGB(2)	JGB(3)	JGB(4)	JGB(5)	JGB(6)	JGB(7)	JGB(8)	JGB(9)	JGB(10)	JGB(11)	JGB(12)
Mean	2402	4009	701	1885	4603	464	380	383	1712	4642	108	111
Median	2057	3714	308	1651	4737	227	177	158	1510	4296	11	31
Maximum	7097	9548	5748	8278	11024	3069	4109	3591	7404	17946	1171	2931
Minimum	0	0	0	0	0	0	0	0	0	0	0	0
Std. Dev.	1408	2002	959	1227	2281	611	576	547	1203	2622	184	234
Skewness	0.801	0.346	2.432	1.304	0.131	1.975	3.168	2.497	1.651	0.938	2.407	7.496
Kurtosis	3.208	2.622	10.365	6.323	2.351	7.099	16.006	10.959	7.113	4.930	9.974	84.585

Table 2 Correlation coefficients between changes in spot rates and BOJ's JGB purchases. The correlation coefficients between d(R(m)) and JGB(n) are presented. Here, d(R(m)) stands for changes in m-year spot rate and JGB(n) stands for BOJ's purchase amount of n-year JGB.

	JGB(1)	JGB(2)	JGB(3)	JGB(4)	JGB(5)	JGB(6)	JGB(7)	JGB(8)	JGB(9)	JGB(10)	JGB(11)	JGB(12)
d(R(1))	－0.038	－0.062	－0.047	－0.104	－0.022	－0.058	0.052	0.070	0.071	0.039	－0.026	－0.021
d(R(2))	－0.004	－0.042	－0.048	－0.063	－0.018	－0.019	0.080	0.100	0.052	－0.017	－0.026	－0.026
d(R(3))	0.007	－0.046	－0.031	－0.035	－0.048	－0.016	0.075	0.127	0.062	－0.049	－0.019	－0.007
d(R(4))	0.026	－0.029	－0.032	－0.026	－0.037	－0.006	0.076	0.122	0.082	－0.039	－0.025	－0.008
d(R(5))	0.011	－0.029	－0.050	－0.049	－0.039	－0.006	0.067	0.108	0.077	－0.055	－0.038	－0.024
d(R(6))	0.006	－0.020	－0.063	－0.049	－0.034	－0.013	0.084	0.130	0.087	－0.060	－0.053	－0.032
d(R(7))	0.025	0.002	－0.075	－0.061	－0.014	－0.011	0.075	0.122	0.078	－0.076	－0.060	－0.049
d(R(8))	0.016	0.017	－0.086	－0.065	－0.008	－0.011	0.080	0.132	0.081	－0.074	－0.050	－0.042
d(R(9))	0.021	0.044	－0.077	－0.054	0.000	－0.007	0.069	0.116	0.084	－0.055	－0.031	－0.034
d(R(10))	0.021	0.050	－0.071	－0.052	－0.006	－0.016	0.038	0.074	0.053	－0.033	－0.002	－0.013
d(R(11))	0.042	0.047	－0.059	－0.043	0.008	－0.008	0.045	0.089	0.058	－0.028	0.009	0.014
d(R(12))	0.035	0.037	－0.062	－0.051	－0.002	－0.005	0.041	0.090	0.064	－0.026	－0.005	0.027

Table 3 Estimates of short rate shochastic processes.

Model I

	Coefficient	Std.Error	z-Statistic	Prob.
θ	0.067371	0.008025	8.394823	0.0000
κ	0.006480	0.000819	7.912166	0.0000
$\ln(\sigma)$	－6.424133	0.022871	－280.880600	0.0000
$\ln(\varepsilon_1)$	－7.431497	0.096048	－77.372810	0.0000
$\ln(\varepsilon_2)$	－7.945550	0.136222	－58.327750	0.0000
$\ln(\varepsilon_3)$	－8.646901	0.106612	－81.106420	0.0000
$\ln(\varepsilon_4)$	－10.286350	0.666176	－15.440880	0.0000
$\ln(\varepsilon_5)$	－8.679653	0.129511	－67.018800	0.0000
$\ln(\varepsilon_6)$	－8.020828	0.215474	－37.224120	0.0000
$\ln(\varepsilon_7)$	－7.512261	0.317087	－23.691450	0.0000
$\ln(\varepsilon_8)$	－7.140090	0.673152	－10.606950	0.0000
$\ln(\varepsilon_9)$	－6.894705	0.849604	－8.115200	0.0000
$\ln(\varepsilon_{10})$	－6.609134	1.095835	－6.031142	0.0000
$\ln(\varepsilon_{11})$	－6.368481	1.363151	－4.671883	0.0000
$\ln(\varepsilon_{12})$	－6.052511	1.156089	－5.235333	0.0000
log likelihood	18718.60			

Model II

	Coefficient	Std.Error	z-Statistic	Prob.
θ	0.044231	0.005181	8.536756	0.0000
κ	0.014042	0.001703	8.247918	0.0000
$\ln(\sigma)$	－6.200587	0.028630	－216.574900	0.0000
λ	0.609534	0.087182	6.991517	0.0000
ψ	0.017553	0.002014	8.715793	0.0000
$\ln(\varepsilon_1)$	－7.339203	0.094574	－77.603000	0.0000
$\ln(\varepsilon_2)$	－7.741975	0.168713	－45.888420	0.0000
$\ln(\varepsilon_3)$	－8.180393	0.164515	－49.724350	0.0000
$\ln(\varepsilon_4)$	－8.769700	0.113710	－77.123630	0.0000
$\ln(\varepsilon_5)$	－10.510830	1.042250	－10.084750	0.0000
$\ln(\varepsilon_6)$	－8.540582	0.107647	－79.338430	0.0000
$\ln(\varepsilon_7)$	－7.898615	0.201516	－39.195900	0.0000
$\ln(\varepsilon_8)$	－7.430269	0.514509	－14.441480	0.0000
$\ln(\varepsilon_9)$	－7.076883	0.746098	－9.485194	0.0000
$\ln(\varepsilon_{10})$	－6.718586	0.885352	－7.588607	0.0000
$\ln(\varepsilon_{11})$	－6.444135	1.389478	－4.637811	0.0000
$\ln(\varepsilon_{12})$	－6.143484	1.188018	－5.171206	0.0000
log likelihood	18928.73			

and shorter maturities JGB yields to 0%.

5 Conclusion

YCC is a novel monetary policy adopted by the BOJ. YCC tries to control not the short-term rate but the term structure of JGB spot rates. The BOJ purchases JGB of various maturities to bring the 10-year JGB yield to zero percent. The effects of BOJ's YCC is measured using a modified version of Jarrow and Li's (2014) model. I find that YCC lowers the term structure of JGB spot rates by 1.7553% per year.

BOJ's JGB holdings are smooth but purchase amount changes rapidly. In this study, I extended the impact $\psi(t,T)$ to a function of BOJ's JGB holdings or purchase amount but could not estimate parameters $\lambda(t,T)$ and $\psi(t,T)$. Therefore, I confirm the assumption of this study that the impact of YCC, $\psi(t,T)$, is an indication variable of whether YCC is effective or not.

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