Is the Supply Curve of Working Hours Theoretically Valid?

— A critical view based on hedonic wage theory —

Tomio Kinoshita[a]

Abstract

This paper argues that the conventional model of working hours has theoretical problems that the hedonic wage model can resolve. The main results are as follows. (1) If the demand curve of working hours is introduced into the system of the conventional model, it becomes overdetermined and incomplete. (2) The conventional model cannot explain market equilibrium in some simple cases, which will mean that it is theoretically incomplete. (3) In the hedonic wage model, the marginal rate of substitution of workers at equilibrium is not equal to the hourly wage rate, and the decreasing effects of income tax on working hours are either smaller or greater than what the conventional model predicts.

JEL Classifications : J22, J31

Keywords : supply curve of working hours, hedonic wage model, bid wage curve, offer wage curve, contract curve

Ⅰ. Introduction

This paper argues that the conventional model of the supply curve of working hours (SCWH) has theoretical problems that the hedonic wage model can resolve. The SCWH is a key topic in the labor economics literature and an important tool for predicting the effects of income tax on working hours. Therefore, the theoretical validity of SCWH is a very important matter.(1)

Much empirical research has been conducted on the SCWH, producing varying estimates of wage rate elasticity. In his survey article, Keane (2011) concluded that there is no clear consensus on its magnitude. This inconclusiveness might be caused by deficiencies in the estimation method, or by theoretical problems with the SCWH. Pencavel (2016) stressed the importance of identifying the demand curve of working hours, arguing that it would be effective in finding the true wage rate elasticity of the SCWH if it were properly built in the estimation model. Contrary to his assertion, we argue that the SCWH has theoretical problems and recommend the hedonic wage model as an alternative framework.

We use a simple equilibrium model that incorporates both working hours and employees. The rest of this paper is organized as follows. Section II outlines the conventional model of the SCWH, and we point out that the model becomes overdetermined and incomplete if the demand curve of working hours is introduced. In section III, we show that this overdetermination is addressed by using the hedonic wage model as indicated by Lewis (1969). In section IV, we compare market equilibriums between the models and argue that the conventional model has problems. Section V compares the models graphically. Section VI compares the implications of the tax effects on working hours. Finally, Section VII offers brief concluding remarks.

Ⅱ. Conventional Model of SCWH

In this section, the conventional model of SCWH is outlined. We point out that if the demand curves of working hours are introduced, the model becomes overdetermined and incomplete. This will suggest that the notion of the supply curve of working hours is defective.

a Musashi University, 1-26-1, Nerima-ku Toyotama Kami, 167-8534, Tokyo Japan.

(1) There are many survey articles on the SCWH. For example, see Killingsworth (1983), Pencavel (1986), Keane (2011) and Bargain and Peich (2013). Broadly speaking, the distinction of the conventional model vs. the hedonic wage model in this paper will correspond to "the fixed wage model vs. the fixed job model" in Trejo (1991) and "the workers-hours demand model vs. the hedonic wage-hours model" in Hart (2004).

Let the utility function of the representative worker be $U(E, t)$, where E is his wage earnings and t is working hours. By definition $E=wt$, where w is the hourly wage rate. In the conventional model, working hours are determined from the workers' utility maximizing behavior as follows:

$Max: U(E, t)$ $st.$ $E=wt.$

The well-known equilibrium condition is

$$-U_t(E, t)/U_E(E, t)=w, \tag{1}$$

which is an implicit expression of SCWH.

Next, let the production function of the representative firm be $F(L, t)$, where L is the number of employees and t is working hours. From its profit maximizing behavior, the demand curves of employees are derived. The profit maximizing behavior is expressed as follows

$$Max\ \pi(L)=F(L, t)-L(wt+C),$$

where C is the fixed employment costs per worker. Here, the price of its output is assumed to be unity for simplicity. The firm controls L given w, C, and t. The equilibrium condition is

$$F_L(L, t)-(wt+C)=0, \tag{2}$$

which is an implicit expression of the demand curve for employees.

Thus, the system of equations is summarized as C1～C4. Equation C4 is the equilibrium condition of the employee's market where the constant $L_{cons.}$ is the supply of employees.

(C1) $(-)U_t(E, t)/U_E(E, t)=w$ (supply curve of working hours)

(C2) $E=wt$ (definition of wage earnings)

(C3) $F_L(L, t)-(wt+C)=0$ (demand curve for employees)

(C4) $L=L_{cons.}$ (equilibrium of the labor market)

The system comprises four endogenous variables (E, t, L, w) and four equations (C1～C4). Therefore, the system is justdetermined and the solution will exist.

The characteristics of the conventional model are:

1 The hourly wage rate (w) has a parametric function, and it is determined from the equilibrium condition of employees.

2 Working hours are determined by workers, and firms will accept it.

3 There is no function that represents firms' demand for working hours in the model.

As Pencavel (2016) claims, if the demand curve of working hours is introduced into the model, it is necessary that working hours (t) be a control variable for the firm. Then, from the profit maximization with respect to working hours, equation (C5) can be formulated.

(C5) $F_t(L, t)-wL=0$ (demand curve for working hours)

Then, the system consists of five equations (C1～C5), while the number of endogenous variables remains four (E, t, L, and w). Therefore, the system becomes overdetermined and incomplete. The reason for overdetermination is that the hourly wage rate must have two roles to equilibrate the markets for both working hours and employees. An approach to resolve the problem was presented by Lewis (1969), which is explained in the next section.

Ⅲ. Hedonic Wage Model

Lewis (1969) presented an innovative model on working hours and wages. He introduced the "employer equalizing wage curve" and the "employee equalizing wage curve" instead of the demand and supply curves of working hours. To put it succinctly, the former is employer's isocost curve (or isoprofit curve) and the latter is employee's indifference curve. His idea was that working hours and wages are determined at the tangency point of these two curves. The idea of Lewis was followed by Rosen (1974). Based on these two papers, Kinoshita (1987) developed the hedonic wage model of working hours.

3. 1　System of Equations

Here we explain the system of equations of the hedonic wage model. We examine the simplest case where all firms have the same technology and all workers have the same preference. The case consists of the following five equations (D1～D5) and five endogenous variables: E (wage earnings), t (working hours), L (number of employees), $\Phi(t)$ (hedonic wage curve) and $Y(E, t)$ (isoprofit curve). C is the fixed employment costs per employee and an exogenous variable.

(D1) $Y_t(E, t)/Y_E(E, t)=U_t(E, t)/U_E(E, t)$
(D2) $E=\Phi(t)$
(D3) $F_L(L, t)-(\Phi(t)+C)=0$
(D4) $F_t(L, t)-L\{d\Phi(t)/dt\}=0$
(D5) $L=L_{cons.}$

The model is explained as follows: Workers pursue utility maximizing behavior under the constraint of the hedonic wage curve $E=\Phi(t)$. The hedonic wage curve $\Phi(t)$ is an endogenous variable and it shows wage earnings of t hour's work in the market equilibrium. The utility maximizing behavior of workers gives the following expression:

Max $U(E, t)$ $\quad$ *st.* $E=\Phi(t)$.

The equilibrium condition is:

$$-U_t(E, t)/U_E(E, t)=d\Phi(t)/dt \tag{3}$$

Next, the firm pursues profit maximizing behavior under the constraint of the hedonic wage curve $E=\Phi(t)$, which gives the following expression:

Max $\pi(L, t)=F(L, t)-L\{\Phi(t)+C\}$,

where the output price is taken as unity for simplicity. Then, the equilibrium conditions are:

$$\pi_L=F_L-\{\Phi(t)+C\}=0 \tag{4}$$
$$\pi_t=F_t-Ld\Phi(t)/dt=0. \tag{5}$$

The isoprofit curve is derived from (4) and (5). It is the solution of a differential equation which is obtained by replacing $\Phi(t)$ with $E(t)$ and eliminating "L." As is explained in the next section, the isoprofit curve is easily derived in the case of Cobb-Douglas type production function.

At the equilibrium point, an indifference curve and an isoprofit curve are tangential to the hedonic wage curve $\Phi(t)$, which leads to equation (D1). The indifference curve and the isoprofit curve at the equilibrium are called the "offer wage curve" and the "bid wage curve" respectively.

Equation (D5) is the condition of the market equilibrium with respect to employees. The demand for employee (L) is given by (D3) which must be equal to the supply of laborers L_{cons}. As the system consists of five equations and five endogenous variables, the system of equations is justdetermined and a solution will exist.(2)

3.2 Bid Wage Curve and the Workers' Equilibrium Condition

We assume a Cobb-Douglas type production function, and derive a bid wage curve. Then the workers' equilibrium condition is derived.

Let us assume the following Cobb-Douglas type production function:

$F(L, t)=AL^{\alpha}t^{\beta}$ (α and β are output elasticities with respect to the number of employees and working hours respectively). Based on equations D3 and D4, the first order conditions are:

$$F_L=\alpha AL^{\alpha-1}t^{\beta}-\{\Phi(t)+C\}=0 \tag{6}$$
$$F_t=\beta AL^{\alpha}t^{\beta-1}-L\,d\Phi(t)/dt=0 \tag{7}$$

From (6) and (7), replacing $\Phi(t)$ with $E(t)$, we obtain the following equation:

$$[E(t)'/\{E(t)+C\}]=(\beta/\alpha)(1/t) \tag{8}$$

The solution of this differential equation is

$$E(t)=kt^{\beta/\alpha}-C, \tag{9}$$

where k is a constant and indicates the profit level. Rewriting (9), we have $(E+C)/t^{\beta/\alpha}=$k. This is the equation of isoprofit curve $Y(E, t)=$k which is used in (D1).(3)

Next, let equation (9) be the bid wage curve. Then, the equilibrium condition of workers is presented from the following maximization problem:

$$\Gamma(E, t, \lambda)=U(E, t)+\lambda[E-(kt^{\beta/\alpha}-C)]. \tag{10}$$

From the first-order conditions, we obtain the following equilibrium condition:

$$(-)\,U_t/U_E=(\beta/\alpha)\{(1+C/E)\}(E/t). \tag{11}$$

This equilibrium condition is very different from that of the conventional model (1). (The marginal rate of substitution is not equal to hourly wage rate $w=E/t$). Furthermore according to DeBeaumont and Singell (1999), the value of (β/α) is likely to be less than 1 and it varies among industries. If this is indeed the case, many

(2) For more details on the hedonic wage model, see Kinoshita (1987).
(3) The smaller the k, the higher the profit level.

estimation results based on the conventional model and its equilibrium condition might produce biased estimates.

3. 3　The Wage-Hour Contract Curve between Firms and Workers

In the hedonic wage model, the equilibrium working hours and wage earnings are determined at the tangential points between a bid wage curve and an offer wage curve. As time goes by, if labor productivity rises, the hedonic wage curve will shift upwards. Accordingly, the equilibrium points will move upwards. The loci of the equilibrium points form a kind of contract curve, which is thus called a wage-hour contract curve. We will obtain wage-hour contract curves from the time series data. On the other hand, we will obtain hedonic wage curves from the cross section data. As wage-hour contract curves and hedonic wage curves are quite different, we must be careful in pooling cross section data and time series data together in empirical works.

The shape of the wage-hour contract curve is determined from the interaction between the indifference curve and the isoprofit curve. It is shown that the familiar substitution effects and income effects (or cost effects) with respect to working hours and wage earnings will determine the loci.[4]

3. 4　Some Implications of the Hedonic Wage Model

The implications of the hedonic wage model are stated as follows:

(1) In the hedonic wage model, hourly wage rate is no longer scalar. Therefore, the notion of the supply curves of working hours is invalid.

(2) In the cross section data, we find the hedonic wage curves. Each labor market will have a different hedonic wage curve.

(3) In the time series data, we find the wage-hour contract curves. The wage-hour contract curve is quite different from the hedonic wage curve. Therefore, we must be careful in pooling the time series and cross section data together in empirical works.

(4) In section 3.1, we assumed that all workers have the same utility function and all firms have the same technology. However, in the general case, each worker will have a different preference and each firm will have a different technology. Then the hedonic wage curve will become a joint envelope of both offer wage curves and bid wage curves (Figure 1).

The shape of the hedonic wage curve is determined by

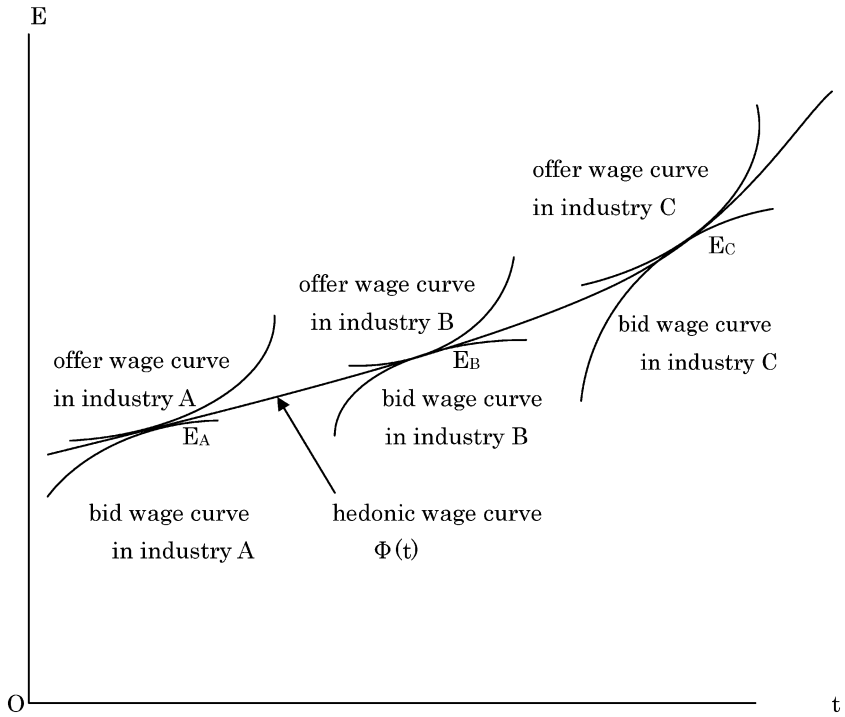

Figure 1.　Hedonic Wage Curve

(4)　For more details, see Kinoshita (1987).

the distribution of both the laborers' preferences as well as the firms' production technologies. This is because at every point on the hedonic wage curve, the supply and demand for laborers must be equated. Therefore, in the general case, the hedonic wage curve will reveal neither the demand structures of firms nor the supply structures of workers.

(5) In general, the hedonic wage curve reveals neither supply nor demand structures. However it must be noted that the hedonic wage curves are increasing function of working hours or $\Phi'(t)>0$ as known from Figure 1. Therefore, if estimation results from the cross section data show $\Phi'(t)<0$, it would indicate that the data were gathered from different labor markets.

Ⅳ. Market Equilibrium

In this section, we compare market equilibrium between the two models. We argue that the conventional model cannot explain market equilibrium adequately. We use a simple model in which there are two types of workers (worker A and worker B) and two types of firms (firm A and firm B). Worker A and worker B have the same quality (productivity) but have different preferences. Firm A and firm B have different technology but hire from the same labor market. Throughout this section, we assume Cobb-Douglas production function $AL^{\alpha}t^{\beta}$. (5)

4. 1 Conventional Model and Market Equilibrium

First, we derive the firms' demand curves for working hours and employees. Their profit maximizing behavior is expressed as follows:

$$\text{Max } \pi(L,\ t)=AL^{\alpha}t^{\beta}-L\ (wt+C)$$

where C is fixed employment costs per worker. Here, the price of output is assumed to be unity. The firm controls L and t given w and C. The equilibrium conditions are

$$\pi_L=A\alpha L^{\alpha-1}t^{\beta}-(wt+C)=0 \tag{12}$$

$$\pi_t=A\beta L^{\alpha}t^{\beta-1}-wL=0 \tag{13}$$

From (12) and (13), we obtains the demand curves for employees and working hours as follows:.

Demand curve for employee:

$$L^{1-\alpha}=A\beta\{C/(\alpha/\beta-1)\}^{\beta-1}(1/w^{\beta}) \tag{14}$$

Demand curve for working hours:

$$t=\{(C)/(\alpha/\beta-1)\}(1/w) \tag{15}$$

Both demand curves are functions of the hourly wage rate. This implies that the hourly wage rate must play two roles in order to equilibrate both markets for employees and working hours, indicating that the conventional model is overdetermined.(6)

Figure 2 illustrates two demand curves of working hours. D_A and D_B are demand curves of firm A and firm B respectively. They are horizontal in the E-t plane, and their height is a constant $C/(\alpha/\beta-1)$ which is determined by their technology. The greater C or the smaller α/β (>1) is, the higher it is. The same figure also shows two supply curves of working hours (S_A and S_B). S_A and S_B are supply curves of worker A and worker B respectively. Under a given hourly wage rate, worker A prefers more earnings with more working hours than worker B.

Market Equilibrium will be achieved at points K and M (not at points J or N). K is the intersection of S_A and D_A, and M is the intersection of S_B and D_B. Obviously, worker A prefers K to N (or prefers firm A to firm B), and that firm A prefers K to J (or prefers worker A to worker B). On the other hand, firm B prefers N to M, but cannot hire worker A, while worker B prefers J to M, but cannot be employed by firm A. Another condition for market equilibrium is that the demand and supply of laborers be equated. The demand curves of firm A and firm B are D_{LA} and D_{LB} respectively, shown on the right-hand side of Figure 2. The supply of laborers to firm A and firm B is indicated by L_A and L_B respectively. When wage earnings are at the level of K and M, the demand and supply of laborers are equal between the firms. Now, let us suppose that the supply of laborers to firm A decrease to L'_A. Then its wage earnings must rise to Q′. However, the conventional model cannot explain the new equilibrium of working hours corresponding to Q′, because the demand curve of working hours does not shift. This illustrates the overdetermination of the con-

(5) The idea of this section owes much to Rosen (1969, pp. 261–263).

(6) The same demand curve of working hours was used in Rosen (1968, p. 517). If $\alpha=\beta$ or labor input is a function of man-hour (Lt), the demand curve of working hours cannot be defined in the conventional model. On the other hand, an isoprofit curve in the hedonic wage model can be defined as $E(t)=kt-C$ from (9).

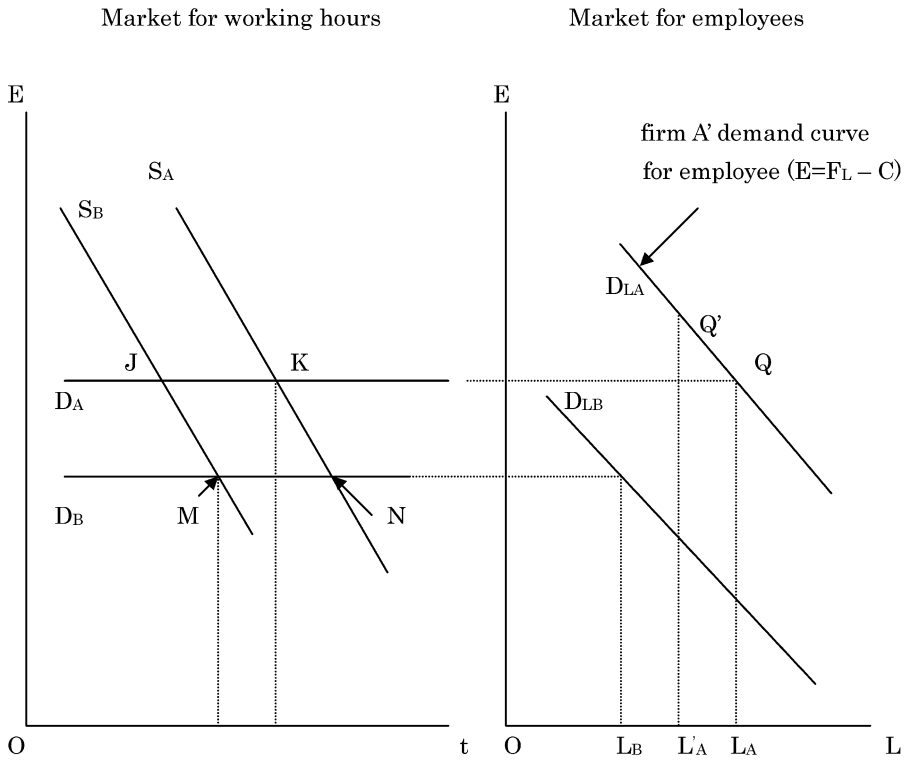

Figure 2.　Conventional Model and its Market Equilibrium

ventional model, which is one of its theoretical problems. This problem is solved by using the hedonic wage model, as explained in the next section.

4. 2　Hedonic Wage Model and Market Equilibrium

The market equilibrium in the hedonic wage model is illustrated in Figure 3. Firm A's equilibrium point is K where its bid wage curve (BWC) is tangent to worker A's offer wage curve (OWC). Similarly, firm B's equilibrium point is M where its BWC is tangent to worker B's OWC. Firm A's demand curve for employees (D_{LA}) is illustrated on the right-hand side of Figure 3. The supply of laborers to firm A is L_A, and its marginal productivity of labor is equal to its wage earnings QL_A.

Now, let us suppose that the supply of laborers to firm A decreases to L^*_A. Then its wage earnings must rise to Q*, and the equilibrium point will move from K to K* along the contract curve. Thus a new market equilibrium will be attained. Whether the equilibrium working hour increases or decreases, it will depend on the slope of the contract curve. It should be stressed here that the slope of the contract curve is not the same as that of the supply curve of working hours in the conventional model.(7)

4. 3　When Workers Have the Same Preference or When Firms Have the Same Technology

How is market equilibrium achieved when workers have the same preferences and firms have different technologies? Figure 4A presents a possibility for the conventional model. K and N are the intersections of the supply and demand curves of working hours, and may be the equilibrium points for firm A and firm B respectively. However, this cannot be market equilibrium, because it is obvious that all workers prefer K to N (as K implies

(7)　The slope of the supply curve of working hours in the conventional model is obtained as follows. Totally differentiating the equilibrium condition (1), we have:

$$dE/dt=[-U_E U_{tt}+U_t U_{Et}]/[U_E U_{tE}-U_t U_{EE}]=\frac{(-U_t/U_E)\times[-U_{tt}+(U_t/U_E)U_{Et}][-1/U_t]}{[U_{tE}-U_t/U_E U_{EE}][1//U_E]}$$

$$=w\times\frac{\text{(income effect on earnings)/(substitution effect on earnings)}}{\text{(income effect on working hours)/(substitution effect on working hours)}}$$

Compare the above equation with the slope of the contract curve given in Kinoshita (1987, p. 1275).

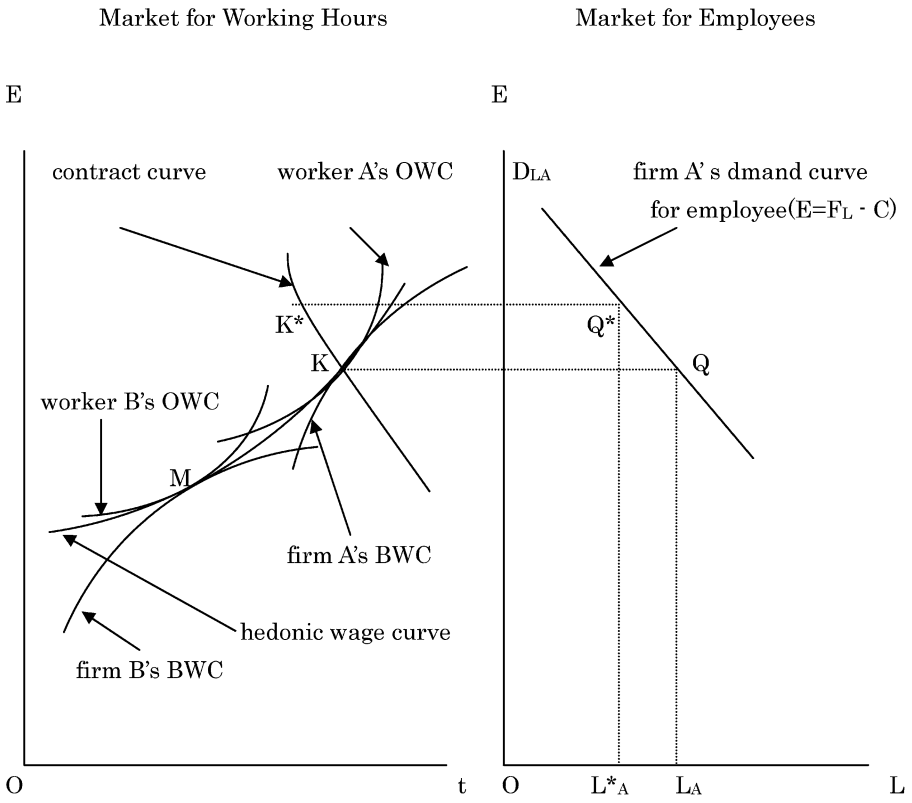

Figure 3. Hedonic Wage Model and its Market Equilibrium

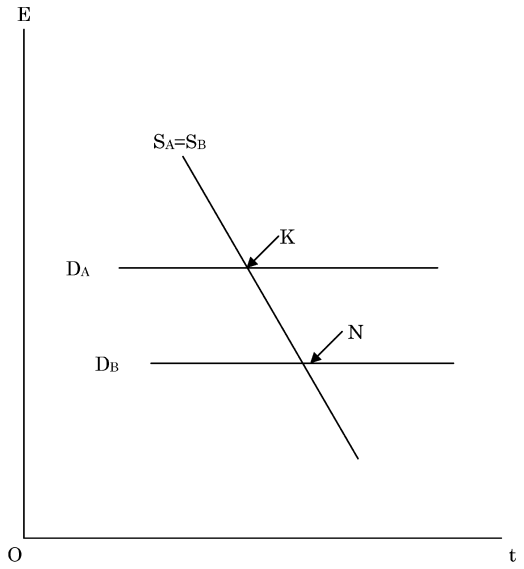

Figure 4A. Conventional Model when Workers Have the Same Preference

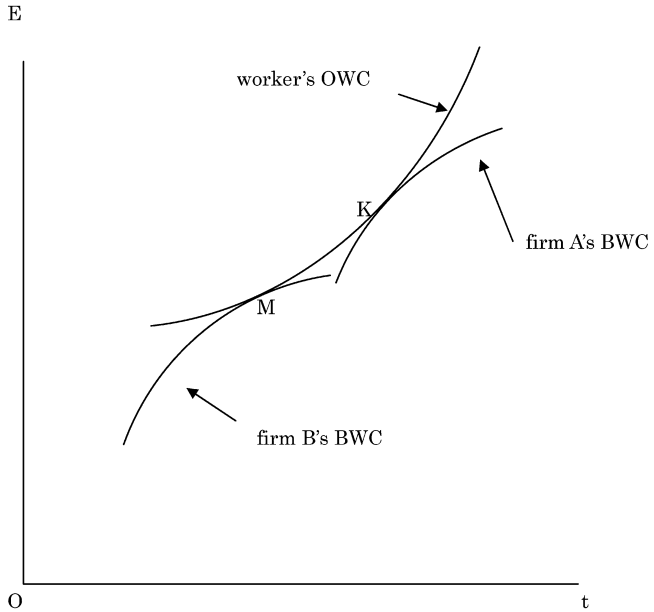

Figure 4B. Hedonic Wage Model when Workers Have the Same Preference

more wage earnings with shorter working hours). Thus the conventional model cannot explain the market equilibrium in this case.

However, the hedonic wage model can explain the market equilibrium in this case easily. In Figure 4B, firm A's equilibrium point is K and that of firm B is M. As K and M are indifferent for workers, this situation can be market equilibrium. The employees in firm A work longer hours and earn higher wages. Here it should be stressed that the hourly wage rate of firm A's worker

(the slope of OK) is either gtreater or smaller than that of firm B's worker (the slope of OM). Therefore, the elasticity of supply of working hours with respect to the hourly wage (between these two workers) is either positive or negative. In other words, the estimates of wage rate elasticity of working hours from cross section data can be either positive or negative.

How is market equilibrium achieved when firms have the same technology and workers have different preferences? Figure 5A shows a possibility for the conventional model. K and J are the intersections of the supply and demand curves of working hours and may be the equilibrium points for firm A and firm B respectively. However, this case cannot be market equilibrium because it is obvious that all workers prefer J to K (as J implies the same wage earnings with shorter working hours). Thus the conventional model cannot explain the market equilibrium in this case.

However, the hedonic wage model can explain the market equilibrium in this case easily. In Figure 5B, K is worker A's equilibrium point and M is that of worker B. For worker A and worker B, K and M are their utility maximizing points respectively. For the firms, K and M are indifferent because they are on the same isoprofit curve. Some firms will choose K, and others will choose M. Therefore, this situation can be market equilibrium. Worker A will work longer hours and earn higher wages. Here it should be stressed that the hourly wage rate of worker A (the slope of OK) is either greater or smaller than that of worker B (the slope of OM). Therefore, the wage rate elasticity of supply of working hours with respect to the hourly wage (between these two workers) is either positive or negative. In other words, the estimates of wage rate elasticity of working hours from cross section data can be either positive or negative.

Ⅴ. Reality of Assumptions and Validity of Theory

What is the crucial difference between the conventional model and the hedonic wage model? The difference is that the assumption of the divisibility of working hours is made in the former but not in the latter. If working hours are divisible, contracts and transactions will be made by the hour, and its price (hourly wage rate) has a parametric function. Thus laborers and firms can choose any number of working hours under a given hourly wage rate, which leads to the effectiveness of supply and demand curves of working hours.

On the other hand, if working hours are indivisible, contracts and transactions are made by the year (e.g. 2000 hours a year) or by indefinite years. In this case, hourly wage rates will be tied to the length of working hours in the contract. Thus, the hourly wage will no longer have a parametric function, and the notions of the supply and demand curve of working hours will not be

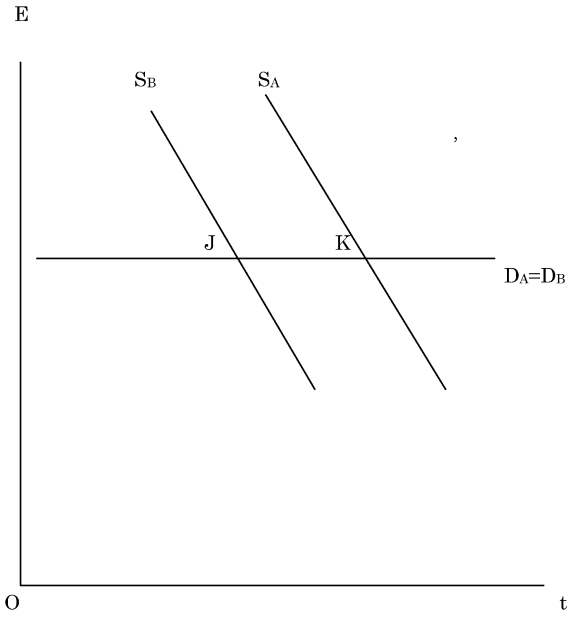

Figure 5A. Conventional Model when Firms Have the Same Technology

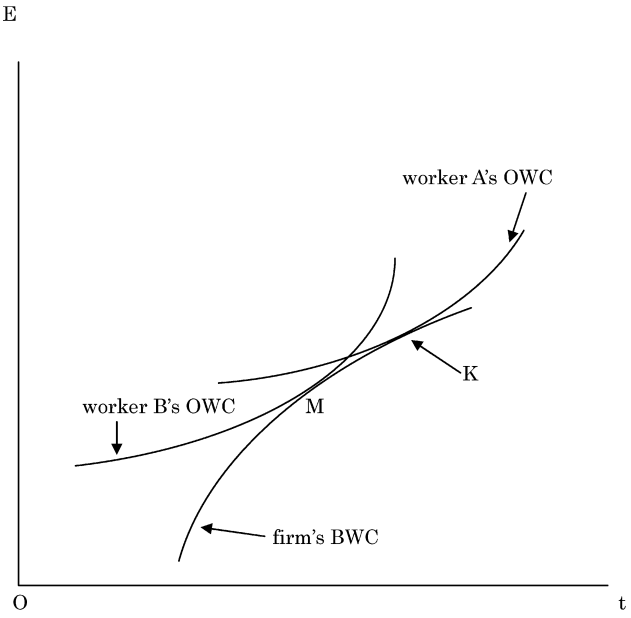

Figure 5B. Hedonic Wage Model when Firms Have the Same Technology

valid. Which assumption is more realistic? The answer is crucial because if the assumption of divisibility is not realistic, estimation results based on the conventional model might not generate accurate results.(8)

5. 1 Graphic Explanation of Demand Curve of Working Hours

In this section, we graphically explain the demand curve of working hours (equation 15) using isoprofit curves in order to facilitate a graphical comparison of the models. Using isoprofit curves (9), the profit maximizing behavior is formulated as follows;

$$\text{Min} \quad k=(E+C)/(t^{\beta/\alpha}) \qquad st.\ E=wt$$

or

$$\text{Min} \quad \Gamma(E,\ t,\ \lambda)=(E+C)/(t^{\beta/\alpha})+\lambda[E-wt]. \qquad (16)$$

From the first-order conditions, we can obtain equation (15). In Figure 6, the demand curve is a horizontal line FG, which is a locus of the tangent points of the isoprofit curves and the cost constraint ($E=wt$).

5. 2 Graphical Explanation of Market Equilibrium of Both Models

In Figure 6, M_C (the intersection of the supply and demand curves of working hours) is the market equilibrium of the conventional model, and its hourly wage rate is the slope OK. However, there is no guarantee that the laborer market is in equilibrium with the hourly wage rate. Suppose that the laborers market equilibrium requires the hourly wage rate OJ. Then, market equilibrium cannot be attained because the firm's equilibrium point is E_D and the worker's equilibrium point is E_S. On the other hand, market equilibrium in the hedonic wage model is M_D (the intersection of the contract curve and OJ). M_D is not on the demand curve of working hours or on the supply curve of working hours. It is only on M_C

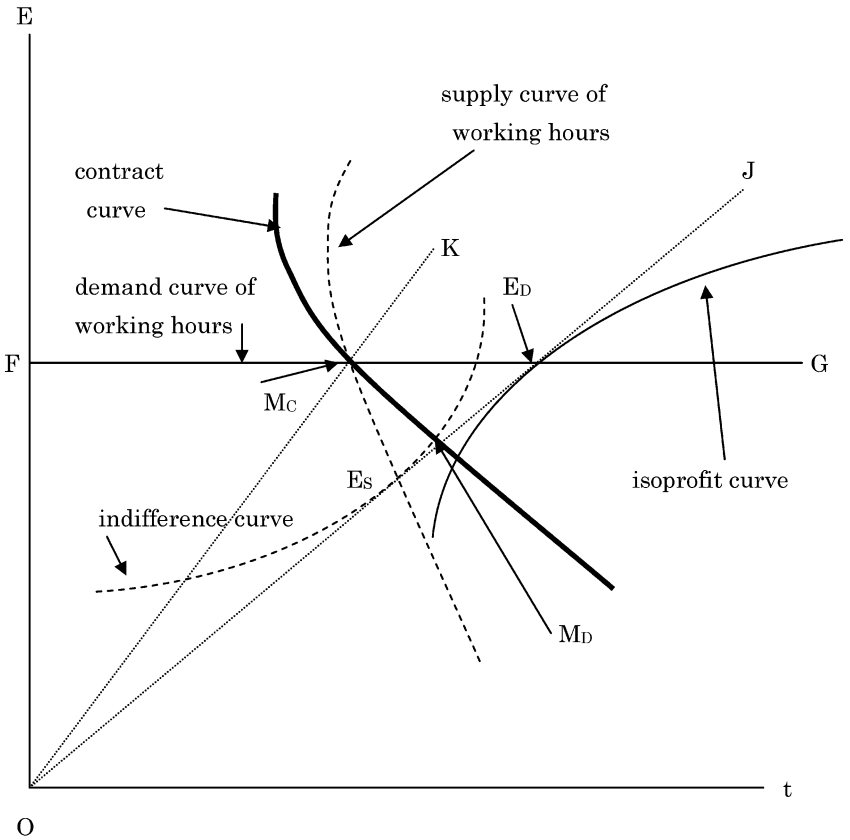

Figure 6. Comparison of Market Equilibrium of both Models

(8) The reality of assumption is not a sufficient condition of the validity of a theory. Friedman (1966) wrote as follows; "Such a theory cannot be tested by comparing its 'assumptions' directly with 'reality.' Indeed, there is no meaningful way in which this can be done. Complete 'realism' is clearly unattainable, and the question whether a theory is realistic 'enough' can be settled only seeing whether it yields predictions that are good enough for the purpose in hand." (p. 41)

that the market equilibrium of the conventional model and the hedonic wage model coincides; the contract curve will pass through M_C because the slope of the indifference curve and isoprofit curve is the same at the point, which is equal to OK. As mentioned, however, the equilibrium hourly wage rate, determined from the laborer market will generally not be equal to OK.

Ⅵ. Tax Effects on Working Hours

In this section, tax effects on working hours are compared using the Slutsky equation. In the hedonic wage model, the bid wage curve becomes the budget constraint for workers.

6. 1 The Conventional Model and Tax Effects

In the conventional model, tax effects on working hours are analyzed as the following maximization problem:

$$\text{Max } U(E,\ t) \qquad st.\ E=(1-\tau)wt,$$

where τ is the tax rate, E is the wage income after tax and w is the hourly wage rate. It is implicitly assumed that workers can choose any number of working hours under a given wage rate.(9)

With the following Lagrangian function,

$$\Gamma(E,\ t,\ \lambda)=U(E,\ t)+\lambda\{E-(1-\tau)wt\},$$

the first-order conditions are as follows

$$U_E+\lambda=0$$

$$U_t-\lambda(1-\tau)w=0$$

$$E-(1-\tau)wt=0.$$

Differentiating these equations with respect to E, t and λ, we obtain the following Slutsky equation:

$$dt/d\tau=(-w/\Delta_C)[U_E+t\{U_{tE}-(U_t/U_E)U_{EE}\}] \qquad (17)$$

where Δ_C is the bordered Hessian and positive. The first term in the parenthesis represents the substitution effects and the second term represents the income effects. If leisure is a normal good and the income effects dominate substitution effects, it follows that $dt/d\tau>0$. Therefore, an increase in tax rate (τ) will increase working hours. In this case, the efficiency loss of taxation will not occur. On the contrary, if the substitution effects dominate the income effects, it follows that $dt/d\tau<0$ and efficiency loss of taxation will occur.(10)

6. 2 The Hedonic Wage Model and Tax Effects

In the hedonic wage model, the budget constraint for workers is a bid wage curve (or a hedonic wage curve), and its slope at the equilibrium point is crucial for the tax effects on working hours. If its slope is the same as the hourly wage, the tax effects will be the same as in the conventional model. But it is possible that the slope of the bid wage curve is either larger or smaller than the hourly wage.

If the slope of the bid wage curve is smaller than the hourly wage (w), let its linear approximation be $I=vt+N$ ($v<w,\ N>0$). Then this line becomes the budget constraint for workers (as $v\to w$ and $N\to 0$, it coincides with the conventional model). The utility maximization problem is expressed as follows:

$$\text{Max } U(E,\ t) \qquad st.\ E=(1-\tau)(vt+N).$$

In the same way as in section 6.1, the Slutsky equation is obtained as follows:

$$dt/d\tau=(-w/\Delta_D)[(v/w)U_E+t\{U_{tE}-(U_t/U_E)U_{EE}\}]. \qquad (18)$$

where Δ_D is the bordered Hessian (see note 9) and $w=I/t$.(11)

Comparing the two Slutsky equations, (17) and (18), it is observed that the magnitude of the income effects are almost the same in both models, but the substitution effects in the hedonic wage model is either smaller or

(9) There are many studies on this topic. For example, see Hausman (1985) and Keane (2011) as well as their references. Keane classifies these models into static and life cycle categories. The former type considers saving and human capital as given, while the latter considers them as control variables.

(10) The bordered Hessian Δ_C and Δ_D are as follows respectively.

$$\Delta_C=\begin{vmatrix} U_{EE} & U_{tE} & 1 \\ U_{tE} & U_{tt} & -(1-\tau)w \\ 1 & -(1-\tau)w & 0 \end{vmatrix},\quad \Delta_D=\begin{vmatrix} U_{EE} & U_{tE} & 1 \\ U_{tE} & U_{tt} & -(1-\tau)v \\ 1 & -(1-\tau)v & 0 \end{vmatrix}$$

(11) From equation (11), the slope of the BWC at the equilibrium point will be known if the values of β/α, C and E/t are given. DeBeaumont and Larry (1999) provide the estimations of α and β for the US industries.

larger (v/w times that of the conventional model). Therefore, the tax effect of decreasing working hours is either smaller or larger than what the conventional model predicts.

Ⅶ. Concluding remarks

The conventional model assumes the divisibility of working hours, and it treats hourly wage rates as parametric variables, whereas the hedonic wage model assumes that working hours are indivisible and treats hourly wage rates as a function of working hours, making the notion of the supply curve of working hours invalid. So few workers hold multiple jobs that the assumption of divisibility seems unrealistic. However, the reality of the assumption is not as important as is the model's predictive or explanatory power using real world data. Concerning this, Keane (2011) claims that the conventional model does not generate satisfactory results.

We argue that introducing the demand curve of working hours to the conventional model would cause a new problem — an overdetermination in its system of equations. In the model, wage rates are required to equilibrate both the markets of the working hours and the number of employees. Moreover the conventional model cannot explain market equilibrium in several simple cases.

Using the hedonic wage model, we obtain hedonic wage curves in the cross section data, and wage-hour contract curves in the time series data. Since these two are quite different, it is recommended that care be taken when pooling cross section and time series data together. The equilibrium point of a worker and a firm lies on the hedonic wage curve, and at the point a firm's BWC and a worker's OWC is tangent to the hedonic wage curve. The equilibrium condition is given by equation (11) or "$-U_t/U_E=(\beta/\alpha)\{(1+C/E)\}(E/t)$," which is different from the conventional model's equilibrium condition of "$-U_t/U_E=(E/t)$." Here "β/α" (the ratio of output elasticity of working hours to employees) is expected to be either smaller or larger than 1 and to differ across industries.

The tax effects on working hours also differ between these two models. In the hedonic wage model, the slopes of the bid wage curve are not equal to hourly wages. Furthermore, the smaller its slope, the smaller the tax effect of decreasing working hours.

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